Mastery of Basic Multiplication Facts for Students With Learning Disabilities
D. Jane Traylor

December 5, 2012

Table of Contents
INTRODUCTION ..... 6
STATEMENT OF PROBLEM ..... 7
REVIEW OF LITERATURE ..... 7
Systematic Explicit Instruction ..... 7
Peer Tutoring ..... 8
Progress Monitoring ..... 9
METHODS ..... 9
Participants ..... 9
Instruments Used ..... 13
Assessment Procedures ..... 14
Teaching Procedures ..... 16
RESULTS ..... 22
Pre-test Results ..... 22
Instructional Results ..... 29
Post Test Results ..... 32
Summary of Results ..... 39
DISCUSSION ..... 40
Teaching Procedures ..... 40
Challenges ..... 41
Supplemental Outcomes ..... 42
Limitations ..... 43
ACKNOWLEDEMENTS ..... 43
REFERENCES ..... 44
APPENDIX A. Instruments Used ..... 45
APPENDIX B. Instructional Instruments ..... 52
APPENDIX C. Progress Monitoring Tools ..... 56

## Table of Figures


Figure 2. Biweekly Two-Minute Multiplication Test Scores -----------------------------------31
Figure 3. Pre- and Post Two-Minute Multiplication Test Scores -------------------------------34
Figure 4. Basic Multiplication Flash Card Pre- and Post Test Scores---------------------------37

## Table of Tables

Table 1. Participant Identifying Information ------------------------------------------------------------10

Table 3. Five-Minute Timed Pre-test Scores -------------------------------------------------------------24
Table 4. Basic Multiplication Flash Card Pre-test Scores -------------------------------------------26
Table 5. Rocket Math Writing Speed Test ------------------------------------------------------------------29
Table 6. Two-Minute Timed Multiplication Post Test Scores -----------------------------------33
Table 7. Basic Multiplication Flash Card Post-Test Scores --------------------------------------36

## Introduction

According to the National Mathematics Advisory Panel [NMAP] (2008), studies of children in the United States consistently show lower achievement in the mastery and fluency of basic math facts when compared to children of many other nations, as well as former U.S. generations. Alarmingly, many U.S. students never reach proficiency in these basic skills, putting them at a disadvantage for performing more complex math. The National Assessment of Educational Progress (2011) reported that nationally only forty percent of Grade 4 students scored at or above proficiency, while thirty-seven percent of Grade 4 students in Oregon scored at or above proficiency in math.

Computational facility with whole number operations rests on the automatic recall of addition and related subtraction facts, and of multiplication and related division facts. It requires fluency with the standard algorithms for addition, subtraction, multiplication, and division. Fluent use of the algorithms not only depends on the automatic recall of number facts but also reinforces it (NMAP, 2008, p. 26).

In a survey of Algebra teachers, the change most requested by these teachers was a more concerted effort at the elementary level in teaching mastery of basic math concepts and skills (NMAP, 2008).

My interest in the student mastery of basic math facts began after volunteering in an elementary school Learning Resource Center (LRC) twice per week from January 2012 to June 2012. I worked with a group of six students in grades three to five who came to the LRC for math. My informal observations over this four-month period showed that these students had few of the basic facts memorized. Rather, they were still working the simple facts, i.e., the plus one facts, minus one facts, and multiplication by two using either finger counting or Touch Math
strategies. In accordance with the findings of the NMAP (2008), this lack of basic math fact automaticity will handicap these students as they move onto more difficult math in middle school. These students, who already have the added struggle of a learning disability, should receive every opportunity to succeed in math, and thus the memorization of the basic math facts.

## Statement of Problem

As stated earlier, only forty percent of Grade 4 students scored at proficiency in math at the national level. There is an obvious need to find a solution to the cause of low U.S. student math scores when compared to those in other nations. A piece to this solution is to apply the proven methods that help students gain proficiency with the basic math facts, a foundational skill for higher-level math.

This study was designed to measure the effects of three research-based strategies used with four Grade 4 and three Grade 5 students with learning disabilities. The key question guiding this study: Would the students be able to develop automaticity with basic multiplication facts as a result of the concurrent implementation of systematic explicit instruction, peer tutoring, and progress monitoring?

## Review of Literature

## Systematic Explicit Instruction

The NMAP (2008) reported, "Explicit systematic instruction was found to improve the performance of students with learning disabilities in computation, solving word problems, and solving problems that require the application of mathematics to novel situations" (p.48). Explicit systematic instruction was described as the teaching of specific strategies followed by opportunities for students to discuss and ask questions about what they were learning. Another aspect that defined explicit systematic instruction was a careful sequencing of the material being
taught with an emphasis placed on key information (NMAP, 2008). "Significant positive effects were also found for Direct Instruction (a specific type of explicit instruction that provides teachers with scripts and that calls for frequent interactions between students and teachers, clear feedback to students on the accuracy of their work, and sequencing of problems so that critical differences are highlighted)" (NMAP, 2008, p 48). Kroesbergen and Van Luit (2003) agreed with the effectiveness of direct instruction and reported that it appeared to be the best method for the teaching of basic math facts.

Baker, Gersten, and Lee (2002) summarized a central recommendation given by the 2001 National Research Council that teacher instruction should play a more active role in helping students make gains in mathematical proficiency. The Council explained that active instruction was essential to having engaged, focused, and involved learners. Thus, systematic explicit instruction is supported by the literature and should be utilized in teaching mathematics.

## Peer Tutoring

Overall, research was quite positive in regards to students studying cooperatively. It appeared that students who worked together as a second phase of instruction were provided a bridge from teacher-directed instruction to independent work. In addition to receiving support in the group work, students benefited by hearing explanations from each other, by having more opportunities to respond, and by receiving additional feedback from each other (Dixon, Carnine, Lee, Wallin, \& Chard, 1998).

Baker et al., (2002) reported that peer-assisted learning interventions consistently produced positive effects on student performance. "Research shows that the use of peers to provide feedback and support improves low achievers' computational abilities and holds promise as a means to enhance problem-solving abilities" (Baker, et al., 2002, pp. 67-68). The NMAP
(2008) concurred that the use of highly structured peer tutoring in the elementary years caused improvement in computation skills.

## Progress Monitoring

Consistently, research showed that mathematics achievement improved when both teachers and students were provided with specific feedback on student performance. This practice has been advocated for many years. The result of such practice is significant, "raising scores, on average, by . 68 standard deviant units" (Baker et al., 2002, p. 67).

The NMAP (2008) stated, "Formative assessment - the ongoing monitoring of student learning to inform instruction - is generally considered a hallmark of effective instruction in any discipline....The average gain in learning provided by teachers' use of formative assessments is marginally significant. Results suggest that the use of formative assessments benefited students at all ability levels" (p. 46).

## Methods

## Participants

The participant group consisted of seven elementary aged students with learning disabilities. There were three Grade 5 students and four Grade 4 students. The study group consisted of two boys and five girls. Table 1 contains each student's age, grade, learning disability, most recent Oregon Assessment of Knowledge and Skills (OAKS) score, and the TwoMinute Timed Basic Multiplication pre-test score.

Table 1
Participant Identifying Information

| Student | Age | Grade | Disability | OAKS Score | TMTM Pre-test |
| :---: | :---: | :---: | :--- | :---: | :---: |
| A | 9 | 4 |  <br> Specific Learning Disability | $193(212)$ | $26 / 100$ |
| B | 9 | 4 | Learning Disability | $206(212)$ | $32 / 100$ |
| C | 9 | 4 | Communication Disorder | $202(212)$ | $18 / 100$ |
| D | 9 | 4 | Specific Learning <br> Disabilities | $203(212)$ | $14 / 100$ |
| E | 11 | 5 | Communication Disorder | $221(219)$ | $25 / 100$ |
| F | 10 | 5 |  <br> Communication Disorder | $219(219)$ | $26 / 100$ |
| G | 10 | 5 | Specific Learning Disability | $208(219)$ | $8 / 100$ |

Note: Oregon Assessment of Knowledge and Skills (OAKS) grade level benchmark scores listed in parentheses following student scores. TMTM = Two-Minute Timed Multiplication Score

Student A. Student A, age nine, was identified with a Language Disorder and Learning Disability on 5/4/2010. During the intervention he was in Grade 4 receiving instruction in the Learning Resource Center (LRC) as follows: 120 minutes per week in Math and 120 minutes per week in Reading and Writing. His IEP math goals included adding/subtracting multi-digit numerals with and without regrouping, learning the 0-9 multiplication facts, and calculating coin values up to five dollars. Socially, Student A is quiet, but friendly, good-hearted, and cooperative. On the Grade 3 OAKS Math test, Student A scored 193, falling below the grade benchmark of 212. Student A correctly responded to 26/100 basic multiplication facts on the Two-Minute Timed Multiplication Pre-test. On the Basic Multiplication Flash Card Assessment,

Student A correctly answered within three seconds the following facts: $0 \mathrm{~s}, 1 \mathrm{~s}$, and 10 s with $100 \%$ accuracy.

Student B. Student B, age nine, was identified with a Learning Disability on 10/27/2011. During the intervention she was in Grade 4, receiving instruction in the LRC as follows: 120 minutes per week in Math. Her IEP math goals address place value to 1000 s, coin values, making change up to one dollar, addition and subtraction with regrouping, word problems, math vocabulary, and skip counting for $2 \mathrm{~s}, 5 \mathrm{~s}, 10 \mathrm{~s}, 3 \mathrm{~s}$, and 4 s . Socially, Student B is cooperative, good-hearted, and friendly. On the Grade 3 OAKS Math test, Student B scored 206 with a grade benchmark of 212. Student B responded correctly to $32 / 100$ basic multiplication facts on the Two-Minute Timed Multiplication Pre-test. On the Basic Multiplication Flash Card Assessment, Student B correctly answered within three seconds the following facts: 0 s and 1 s with $100 \%$ accuracy, 2 s with $90 \%$ accuracy, and 5 s with $80 \%$ accuracy.

Student C. Student C, age nine, was identified with a Communication Disorder on $4 / 29 / 2010$. During the intervention she was in Grade 4 receiving instruction in the LRC as follows: 120 minutes per week in Math, 90 minutes per week in Reading, and 30 minutes per week in Writing. Her IEP math goals address identifying types of triangles, interpreting data on graphs, understanding range, mode, and mean, conceptual understanding of fractions, and story problems. Socially, Student C is friendly, social, follows directions, and has a positive attitude. On the Grade 3 OAKS Math test, Student C scored 202 with a grade benchmark of 212 . Student C responded correctly to $18 / 100$ basic multiplication facts on the Two-Minute Timed Multiplication Pre-test. On the Basic Multiplication Flash Card Assessment, Student C correctly answered within three seconds the following facts: 0 s with $100 \%$ accuracy, 1 s and 2 s with $90 \%$ accuracy, and 5 s with $80 \%$ accuracy.

Student D. Student D, age nine, was identified with Specific Learning Disabilities on $5 / 25 / 2012$. During the intervention she was in Grade 4 receiving instruction in the LRC as follows: 180 minutes per week in Reading and Written Language and 180 minutes per week in Math. Her IEP math goals address learning the multiplication facts through the 10s, using models to solve problems, perimeter, applying models of division, and common fractions. Socially, Student D is kind, friendly, has a good attitude, and enjoys interactions with classmates and adults. On the Grade 3 OAKS Math test, Student D scored 203 with a grade benchmark of 212. Student D correctly responded to $14 / 100$ basic multiplication facts on the Two-Minute Timed Multiplication Pre-test. On the Basic Multiplication Flash Card Assessment, Student D correctly answered within three seconds the following facts: 1 s and 2 s with $90 \%$ accuracy.

Student E. Student E, age eleven, was identified with a Communication Disorder on 6/16/2010. During the intervention he was in Grade 5 receiving instruction in the LRC as follows: 60 minutes daily for Reading and Written Language and 30 minutes daily for Math. His IEP math goals address fractions, math facts ( $3 \mathrm{~s}, 4 \mathrm{~s}, 6 \mathrm{~s}, 8 \mathrm{~s}$ ), and story problems. Socially, Student E is a sweet young man who is friendly, kind, and at times immature for his age. On the Grade 4 OAKS Math test, Student E scored 221 meeting the grade benchmark of 219. Student E correctly responded to $25 / 100$ basic multiplication facts on the Two-Minute Timed Multiplication Pre-test. On the Basic Multiplication Fact Card Assessment, Student E correctly answered within three seconds the following facts: $0 \mathrm{~s}, 1 \mathrm{~s}$, and 2 s with $100 \%$ accuracy; $4 \mathrm{~s}, 5 \mathrm{~s}$, and 9 s with $90 \%$ accuracy; 3 s with $80 \%$ accuracy.

Student F. Student F, age ten, was identified with a Communication Disorder and Specific Learning Disorder on 2/9/2011. During the intervention she was in Grade 5 and receiving instruction in the LRC as follows: 30 minutes per day in Math and 35 minutes per day
in Reading. Her IEP math goals address adding/subtracting fractions, multiplication facts 0-9, multiplying two and three digit numbers, long division, identifying types of angles, solving basic one-step word problems, and making change from $\$ 10$. Socially, Student F is hard working, conscientious, careful, polite, compliant, and tends toward being a perfectionist. On the Grade 4 OAKS Math test, Student F scored 219 meeting the grade benchmark of 219 . Student F correctly responded to 26/100 basic multiplication facts on the Two-Minute Timed Multiplication Pre-test. On the Basic Multiplication Flash Card Assessment, Student F correctly answered within three seconds the following facts: 0 s and 1 s with $100 \%$ accuracy; 5 s with $90 \%$ accuracy.

Student G. Student G, age ten, was identified with a Specific Learning Disability on $5 / 12 / 2012$. During the intervention she was in Grade 5 receiving instruction in the LRC as follows: 120 minutes per week in Math, 105 minutes per week in Reading, and 35 minutes per week in Written Language. Her IEP math goals address area, perimeter, two-digit by two-digit multiplication, fraction computation, decimals, and word problems. Socially, Student G is diligent, helpful, considerate, hardworking, friendly, and kind. On the Grade 4 Math OAKS test, Student G scored 208 falling below the grade benchmark of 219 . Student $G$ correctly responded to $8 / 100$ basic multiplication facts on the Two-Minute Timed Multiplication Pre-test. On the Basic Multiplication Flash Card Assessment, Student G correctly answered within three seconds the following facts: 1 s and 2 s with $90 \%$ accuracy.

## Instruments Used

Untimed addition/subtraction quiz. The Untimed Addition/Subtraction Quiz contained ten addition problems and ten subtraction problems (Appendix A). The addition portion consisted of ten single digit problems with sums ranging from eight to sixteen. The subtraction quiz contained one- and two-digit minuends ranging from eight to eighteen and one-digit
subtrahends with differences ranging from two to nine. A range of problems was selected to test student's basic addition and subtraction strategies.

Two-minute/five-minute timed addition test. The Two-Minute/Five-Minute Addition Test contained 100 single digit addition problems with sums ranging from zero to twenty. Scores were recorded at two and five minute intervals (Appendix A).

Two-minute/five-minute timed subtraction test. The Two-Minute/Five-Minute Subtraction Test contained 100 basic subtraction problems with minuends ranging from one to eighteen, one-digit subtrahends, and differences ranging from zero to nine. Scores were recorded at two and five minute intervals (Appendix A).

Two-minute/five-minute timed multiplication test. The Two-Minute/Five-Minute Multiplication Test contained 100 basic multiplication problems with one-digit factors and products ranging from zero to eighty-one. Scores were recorded at two and five minute intervals (Appendix A).

Basic multiplication flash card assessment. The Basic Multiplication Flash Card Assessment consisted of 100 flash cards containing single digit factors for the zero to nine facts. The test began with the ten zero facts and progressed through the nine facts. The flash cards for each fact family were presented out of order.

Writing speed assessment from rocket math. The Writing Speed Assessment from Rocket Math contained rows of boxes containing a number in the top left of each box (Appendix A). The test contained a total of 42 boxes into which the numbers were to be copied.

## Assessment Procedures

Pre-test procedures. The students were given six different tests to determine baseline information both for the beginning of the school year and for this intervention.

Untimed addition/subtraction quiz. This written test was administered in a group setting. It was given to assess whether the students had an effective strategy for working addition \& subtraction problems.

Two-minute/five-minute timed addition test. This written test was administered in a group setting. The students' scores were recorded at two minutes and at five minutes. This test was requested by the teacher to provide baseline information at the beginning of the school year.

Two-minute/five-minute timed subtraction test. This written test was administered in a group setting. The students' scores were recorded at two minutes and at five minutes. This written test was requested by the teacher to provide baseline information at the beginning of the school.

Two-minute/five-minute timed multiplication test. This written test was administered in a group setting. The students' scores were recorded at two minutes and at five minutes. The twominute test score was used to provide baseline information for the intervention.

Basic multiplication flash card assessment. This oral test was administered individually. It was given to assess the student's automaticity with the zero to nine basic multiplication facts. Students were asked to read the problem on each flash card and to give the answer. If the students gave the answer within three seconds, it was considered correct. Students were shown the entire 100 flash cards regardless of how many were missed. The tester sorted the flash cards into piles of missed and correct answers. This was hidden behind a desktop partition, so students were unaware of this process. Correct answers were recorded on a Multiplication Checklist after the testing session (Appendix A).

Rocket math writing speed assessment. This written test was administered in a group setting. Students were given one minute to write the numbers contained in the 42 boxes on the
test. The amount of numbers copied determined the students' beginning writing speed for daily timed tests.

Instructional procedures. During the weeks of instruction, the students were given daily one-minute timed tests and biweekly Two-Minute Timed Tests.

Daily one-minute timed tests. This written test was administered in a group setting. The one-minute timed tests (Appendix B) were a component of the Rocket Math Program, which is described in the Teaching Procedures section of this paper. The test consisted of forty problems. Newly introduced facts appeared on the test at least twice. The remaining problems were a mix of previously learned facts.

Two-minute timed multiplication test. This same test was used for pre-testing. The TwoMinute Timed Multiplication Test was a written test administered in a group setting every two weeks to track progress. It contained 100 basic multiplication problems with one-digit factors and products ranging from zero to eighty-one.

Post-test procedures. The post testing included the written Two-Minute Timed Multiplication Test administered in a group setting and the Basic Multiplication Flash Card Assessment administered individually.

## Teaching Procedures

The intervention incorporated three concurrent methods: explicit systematic instruction, peer tutoring, and progress monitoring, aimed at improving the mastery and fluency of basic multiplication facts. As this study involved Grade 4 and Grade 5 students with learning disabilities, multiplication was chosen as the focus.

We recommend that multiplication and division facts be presented to intermediate grade remedial students before addition and subtraction. The reason for this recommendation is
that these students are likely to have some type of finger strategy that allows them to compute addition and subtraction facts correctly. On the other hand, these students are likely to have no viable strategy for figuring out multiplication and division facts (Stein, Kinder, Silbert, and Carnine, 2006, p. 82).

The students in this intervention were given an untimed Addition/Subtraction Quiz to ascertain whether they possessed adequate strategies for performing addition and subtraction. Those results will be discussed in the Pre-test/Baseline section of this paper.

The intervention took place during the students' 30 -minute math class in the (LRC). The Grade 4 students attended class five days per week, while the Grade 5 students attended four days per week. The intervention was performed in conjunction with other math instruction. In the early weeks of the implementation, extra time was spent training the students in the program. The optimum goal for completing the math facts intervention was ten minutes.

In the initial meeting with the mentor teacher, we learned that one of the district's approved math fact programs would need to be used, Carnine Math or Rocket Math (1998). Since the teacher's preference was for Rocket Math, its program became an integral part of the intervention.

Rocket math description. Rocket Math contained two initial assessments: The Writing Speed Test and the Two-Minute Timed Test. The Writing Speed Test was used to determine the number of problems a student could be expected to complete in one minute. The Two-Minute Timed Test was created for progress monitoring on a weekly basis. An optional assessment included in Rocket Math was Placement Probes, 15-second mini-tests, to allow students to skip some of the sets if the facts were previously memorized. Other components of the program included:

- Goal Sheets for recording a student's writing speed
- One-Minute Practice/Test Sheets
- Individual Rocket Charts for recording daily scores and progress
- Individual Student Graphs for recording Two-Minute Timed Test Scores
- Peer Tutoring.

The multiplication piece of the program contained worksheets labeled Set A-Set W (Appendix B). With the exception of the ones and zero facts, each worksheet introduced no more than two new facts and their reverses per sheet. See Appendix B for the multiplication fact sequence. Each worksheet was divided in half with forty problems on the top used for practice and forty problems on the bottom used for the one-minute timed test. Rocket Math allowed students to progress to the next practice/test sheet if they met or beat their previous writing goal. For students capable of completing forty problems, thirty-eight problems answered correctly were necessary to pass to the next set.

Rocket Math used peer tutoring for worksheet practice. Students were put into pairs with one student as the tutor and one as the "student." The student was instructed to read each problem aloud and to give the answer. The tutor followed an answer sheet and was required to give an immediate correction for a hesitation or an error. The student was directed to say the missed problem with the answer three times and to go back three problems to begin again. After two to three minutes, the students switched roles. After peer tutoring was completed, Rocket Math had the students take the One-Minute Timed Test.

The daily intervention components were ordered as follows: (a) Systematic explicit teaching, (b) Peer tutoring, (c) Timed tests, and (d) Progress Monitoring. In order to concentrate
on teaching the program components, especially the peer tutoring piece, all the students were started on Set A.

Systematic explicit instruction. Even though all the students began on Set A, they progressed at different paces making systematic explicit teaching a challenge. Small dry erase boards were used to introduce each student's new facts for the day and were prepared ahead of students' arrival. The facts were written with the answers. The student was required to say the problem with the answer. The answers were erased, and the student said the problem and supplied the answer. Then, the entire problem was erased requiring the student to say the problem with the answer from memory. Corrections were given immediately and the student was required to repeat the problem and answer correctly. For the first several weeks of the intervention, flash cards were used to review the sets previously learned. However, the flash card review was dropped after Week 3, since the students were receiving review during the peertutoring component.

Peer tutoring. As soon as students received systematic explicit instruction on their math facts for the day, they were paired off for peer tutoring. During the first several weeks of the intervention extra time was spent training the students in the peer-tutoring method as described in the Rocket Math Description section of this paper. A tabletop poster (Appendix B) was used to review the tutoring methods before each session until the students became proficient with the method. Peer tutoring was modeled for the students with special attention given to defining $a$ hesitation. Tutors were trained to ask the student what set they were on, and then to immediately find the corresponding set in the answer key. The tutor then gave the cue to begin. Tutors were coached to use their fingers to track in the answer key as the student stated the problems with the answers. They were taught to immediately stop a student who gave a wrong answer or who
hesitated for more than a second. The tutor would instruct the student to repeat the missed problem three times with the correct answer. Then, the student was instructed to go back three problems and begin again. After the first week, the students knew the process and had no need for repeated directions on what to do. The students were closely monitored during the initial training to insure they were tutoring correctly. The students were given one minute to practice with their tutor and then were told to switch positions, allowing their partner to practice for one minute. It was necessary to limit each student to one minute of practice in order to streamline the intervention to ten-minutes.

Progress monitoring. After the peer tutoring session, the students took a one-minute timed test. After the students' papers were corrected, they recorded the number correct on their Individual Rocket Graphs (Appendix C). Beside each set were spaces to record up to ten trials. If the student passed the set, he/she colored in a segment for that set on their rocket. Every two weeks the students were given the same Two-Minute Timed Basic Multiplication Test. These scores were recorded on an individual graph (Appendix C). Students colored a bar graph to match their score for each test allowing them to monitor their progress. A magnetic bulletin board (Figure 1) was created to allow students to move a rocket with their name on it as they progressed through the sets.

Figure 1. Magnetic Bulletin Board


Figure 1. The Magnetic Bulletin Board was used as an incentive for progress. The letters at the bottom of the board coordinated with the multiplication sets. Students moved their individual rockets each time they passed a set.

Data collection procedures. Students recorded their attendance on a "Bee on Time" Chart (Appendix D). The chart had a bee themed border in keeping with the theme of the school's Positive Behavior Support Plan. Students checked a box beside their name and under the date for every day they came to class on time. The teacher recorded absences on the chart and the reason for the absence.

Individual Student Graphs (Appendix C) were used to record daily scores on the OneMinute Timed Tests, as well as the number of trials required to pass a level. Student scores for the biweekly Two-Minute Timed Tests were recorded and graphed by the students.

Incentives. It became clear early on that the students had to get to class on time. Thus, an incentive was introduced using the school's "Bee System" for positive behavior. Students were introduced to a "Bee on Time Chart" (Appendix C) where they would sign in upon arrival to
class, earning ten points per day. After collecting two hundred points, the students were able to earn a prize. This system was amazingly effective. Students were rarely late for class and were generally a few minutes early.

The students were also able to earn a prize for every five of the multiplication sets they passed. The prizes included used children's books, special pencils, rubber toys, markers, and small stuffed animals.

Staff training. The special education aide for the LRC was trained in each part of the intervention. Each component of the program was explained to her. The aide spent time during the first week observing the program being administered. She received specific information on peer tutoring concerning the correct way to give corrections. The aide ran the program on Fridays for the Grade 4 students.

## Results

## Pretest Results

Prior to beginning the intervention, each student was given an Untimed Addition Quiz (UAQ), Untimed Subtraction Quiz (USQ), Two-Minute Timed Addition Test (TMTA), TwoMinute Timed Subtraction Test (TMTS), and a Two-Minute Timed Multiplication Test (TMTM). Table 2 contains the students' scores on those tests. The average number of seconds per problem (ANSM) on the Two-Minute Timed Multiplication Test was calculated for each child.

Table 2
Pre-test Scores

| Student | UAQ | USQ | TMTA | TMTS | TMTM | ANSM |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | $6 / 10$ | $3 / 10$ | $30 / 100$ | $41 / 100$ | $26 / 100$ | 4.6 |
| B | $10 / 10$ | $10 / 10$ | $46 / 100$ | $49 / 100$ | $32 / 100$ | 3.8 |
| C | $10 / 10$ | $10 / 10$ | $47 / 100$ | $47 / 100$ | $18 / 100$ | 6.7 |
| D | $9 / 10$ | $10 / 10$ | $53 / 100$ | $52 / 100$ | $14 / 100$ | 8.6 |
| E | $10 / 10$ | $10 / 10$ | $26 / 100$ | $17 / 100$ | $25 / 100$ | 4.8 |
| F | $10 / 10$ | $10 / 10$ | $44 / 100$ | $32 / 100$ | $25 / 100$ | 3.8 |
| G | $10 / 10$ | $10 / 10$ | $40 / 100$ | $36 / 100$ | $8 / 100$ | 12.5 |

Note: UAQ = Untimed Addition Quiz; USQ = Untimed Subtraction Quiz; TMTA = TwoMinute Timed Addition Test; TMTS = Two-Minute Timed Subtraction Test; TMTM = Two-Minute Timed Multiplication Test; ANSM = Average Number of Seconds Per Problem on Multiplication Test. Tests administered September 17-19, 2012.

The students were given Five-Minute Timed Addition Tests (FMTA), Five-Minute Timed Subtraction Tests (FMTS), and Five-Minute Timed Multiplication Tests (FMTM) to provide additional baseline information requested by the teacher. This additional information can be found below in Table 3.

Table 3
Five-Minute Timed Pre-test Scores

| Student | FMTA | FMTS | FMTM |
| :---: | :---: | :---: | :---: |
| A | $47 / 100$ | $64 / 100$ | $36 / 100$ |
| B | $97 / 100$ | $91 / 100$ | $69 / 100$ |
| C | $88 / 100$ | $83 / 100$ | $42 / 100$ |
| D | $99 / 100$ | $97 / 100$ | $32 / 100$ |
| E | $54 / 100$ | $62 / 100$ | $58 / 100$ |
| F | $87 / 100$ | $66 / 100$ | $52 / 100$ |
| G | $72 / 100$ | $66 / 100$ | $18 / 100$ |

Note: FMTA = Five-Minute Timed Addition Test; FMTS = Five-Minute
Timed Subtraction Test; FMTM = Five-Minute Timed Multiplication Test
Two-minute timed basic multiplication test. The Two-Minute Timed Multiplication Test contained 100 problems with one-digit factors and products ranging from zero to eightyone. At the time of pre-testing, not all of the Rocket Math components were available, so the Two-Minute Timed Multiplication Test used in the pre-test was from Carnine Math.

Student A. Student A correctly answered 26 of 100 problems and demonstrated understanding of the following facts: $0 \mathrm{~s}, 1 \mathrm{~s}, 2 \mathrm{~s}$, and 5 s . He averaged 4.6 seconds per problem on the test demonstrating weakness in automaticity of the basic facts. Student A answered two problems incorrectly: $3 \times 1=4 ; 3 \times 2=15$.

Student B. Student B correctly answered 32 of 100 problems and demonstrated understanding of the following facts: $0 \mathrm{~s}, 1 \mathrm{~s}, 2 \mathrm{~s}$, and 5 s . She averaged 3.8 seconds per problem on
the test demonstrating weakness in automaticity of the basic facts. Student B answered one problem incorrectly: $5 \times 5=20$.

Student C. Student C correctly answered 18 of 100 problems and demonstrated understanding of the following facts: $1 \mathrm{~s}, 2 \mathrm{~s}$, and 5 s . She averaged 6.7 seconds per problem on the test demonstrating significant weakness in automaticity of the basic facts. Student C showed confusion with zero facts: $0 \times 1=1 ; 0 \times 6=6$.

Student D. Student D correctly answered 14 of 100 problems and demonstrated understanding of the following facts: 2 s and 5 s . She averaged 8.6 seconds per problem on the test demonstrating weakness in automaticity of the basic facts. Student D showed confusion with zero facts by answering with the number being multiplied by zero, i.e., $0 \times 1=1$.

Student E. Student E correctly answered 25 of 100 problems and demonstrated understanding of the following facts: $0 \mathrm{~s}, 1 \mathrm{~s}, 2 \mathrm{~s}, 3 \mathrm{~s}, 4 \mathrm{~s}$, and 5 s . He averaged 4.8 seconds per problem on the test demonstrating weakness in automaticity of the basic facts. Student E did not make any errors on the problems answered. Processing issues due to his learning disability slow down Student E's performance.

Student F. Student F correctly answered 25 of 100 problems and demonstrated understanding of the following facts: $0 \mathrm{~s}, 1 \mathrm{~s}, 2 \mathrm{~s}, 3 \mathrm{~s}, 4 \mathrm{~s}$, and 5 s . She averaged 3.8 seconds per problem on the test demonstrating weakness in automaticity of the basic facts. Student F did not make any errors on the problems answered. It is suspected that her perfectionist tendencies with writing slowed down her performance.

Student G. Student G correctly answered 8 of 100 problems and demonstrated understanding of the following facts: 2 s . She averaged 12.5 seconds per problem on the test
demonstrating extreme weakness in automaticity of the basic facts. Student G responded to 17 problems incorrectly. Student G showed the following errors: answered ones facts with one on five problems; answered five facts incorrectly by adding five to the answer, i.e., $5 \times 4=25$, $5 \times 5=30$, and $3 \times 4=20$.

## Basic multiplication flash card assessment. On the Basic Multiplication Flash Card

 Assessment students were shown 100 flash cards containing the zero to nine multiplication facts to assess the students' automaticity with these basic facts. Students were asked to read the problem on the flash card and to give the answer. After reading the problem, if the student responded correctly within three seconds, the answer was considered correct. The scores for the Basic Multiplication Flash Card Assessment Pre-test scores are found in Table 4.Table 4
Basic Multiplication Flash Card Pre-test Scores

| Student | 0 s | 1 s | 2s | 3 s | 4 s | 5s | 6s | 7 s | 8s | 9 s |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | 10/10 | 10/10 | 5/10 | 6/10 | 5/10 | 10/10 | 3/10 | 2/10 | 2/10 | 2/10 |
| B | 10/10 | 10/10 | 9/10 | 7/10 | 7/10 | 8/10 | 7/10 | 4/10 | 4/10 | 3/10 |
| C | 10/10 | 9/10 | 9/10 | 6/10 | 6/10 | 8/10 | 3/10 | 4/10 | 3/10 | 2/10 |
| D | 1/10 | 9/10 | 9/10 | 2/10 | 0/10 | 2/10 | 2/10 | 0/10 | 1/10 | 3/10 |
| E | 10/10 | 10/10 | 10/10 | 8/10 | 9/10 | 9/10 | 6/10 | 6/10 | 5/10 | 9/10 |
| F | 10/10 | 10/10 | 7/10 | 7/10 | 6/10 | 9/10 | 3/10 | 3/10 | 3/10 | 2/10 |
| G | 1/10 | 9/10 | 9/10 | 5/10 | 6/10 | 6/10 | 3/10 | 3/10 | 4/10 | 3/10 |

Any answer requiring more than 3 seconds was considered incorrect. Bold type indicates facts not mastered with $80 \%$ or above automaticity.

Student A. Student A demonstrated automaticity by responding within three seconds to the following facts: $0 \mathrm{~s}, 1 \mathrm{~s}$, and 5 s with $100 \%$ accuracy. Student A showed limited automaticity and lack of memorization as indicated by the following results: 3 s and 6 s with $60 \%$ accuracy; 2 s and 4 s with $50 \%$ accuracy; and 7 s, 8 s, and 9 s with $20 \%$ accuracy.

Student B. Student B demonstrated automaticity by responding within three seconds to the following facts: 0 s and 1 s with $100 \%$ accuracy; 2 s with $90 \%$ accuracy; and 5 s with $80 \%$ accuracy. Student B showed limited automaticity and lack of memorization as indicated by the following results: $3 \mathrm{~s}, 4 \mathrm{~s}$, and 6 s with $70 \%$ accuracy; 7 s and 8 s with $40 \%$ accuracy; and 9 s with $30 \%$ accuracy.

Student C. Student C demonstrated automaticity by responding within three seconds to the following facts: 0 s with $100 \%$ accuracy; 1 s and 2 s with $90 \%$ accuracy; and 5 s with $80 \%$ accuracy. Student C showed limited automaticity and lack of memorization as indicated by the following results: 3 s and 4 s with $60 \%$ accuracy; 7 s with $40 \%$ accuracy; 6 s and 8 s with $30 \%$ accuracy; and 9 s with $20 \%$ accuracy.

Student D. Student D demonstrated automaticity by responding within three seconds to the following facts: 1 s and 2 s with $90 \%$ accuracy. Student D showed limited automaticity and lack of memorization as indicated by the following results: 9 s with $30 \%$ accuracy; $3 \mathrm{~s}, 5 \mathrm{~s}$, and 6 s with $20 \%$; 0 s and 8 s with $10 \%$ accuracy.

Student E. Student E demonstrated automaticity by responding within three seconds to the following facts: $0 \mathrm{~s}, 1 \mathrm{~s}$, and 2 s with $100 \%$ accuracy; $4 \mathrm{~s}, 5 \mathrm{~s}$, and 9 s with $90 \%$ accuracy; and 3 s with $80 \%$ accuracy. Student E showed limited automaticity and lack of memorization as indicated by the following results: 6 s and 7 s with $60 \%$ accuracy; and 8 s with $50 \%$ accuracy.

Student $\boldsymbol{F}$. Student F demonstrated automaticity by responding within three seconds to the following facts: 0 s and 1 s with $100 \%$ accuracy; and 5 s with $90 \%$ accuracy. Student F showed limited automaticity and lack of memorization as indicated by the following results: 2 s and 3 s with $70 \%$ accuracy; 4 s with $60 \%$ accuracy; $6 \mathrm{~s}, 7 \mathrm{~s}$, and 8 s with $30 \%$ accuracy; and 9 s with $20 \%$ accuracy. Student F showed automaticity on 60 of 100 problems.

Student G. Student G demonstrated automaticity by responding within three seconds to the following facts: 1 s and 2 s with $90 \%$ accuracy. Student G showed limited automaticity and lack of memorization as indicated by the following results: 4 s and 5 s with $60 \%$ accuracy; 3 s with $50 \%$ accuracy; 8 s with $40 \%$ accuracy; $6 \mathrm{~s}, 7 \mathrm{~s}$, and 9 s with $30 \%$ accuracy; and 0 s with $10 \%$ accuracy. Student G showed automaticity on 49 of 100 problems.

Rocket math writing speed test. The Rocket Math Writing Speed Test was used to establish writing speed goals for students on the daily One-Minute Timed Tests. According to Rocket Math (2007), "many children are not able to write the answers to 40 problems in one minute." Goals are established for students that are no faster than they are able to write. For example, if a student scored 29 on the speed test, a student could progress to the next fact set if he/she answered 29 problems correctly on his/her current math set. Table 5 contains the beginning writing speed for each student. The highest writing speed possible was 42 . The table shows that Student E had a substantially slower writing speed than the other students. Student F showed a slightly slower speed. A student's writing speed increased as they surpassed their current writing speed. After several weeks, Students E and F increased their writing speeds to 40 .

Table 5
Rocket Math Writing Speed Test

| Student | Writing Speed |
| :---: | :---: |
| A | 42 |
| B | 42 |
| C | 42 |
| D | 42 |
| E | 29 |
| F | 39 |
| G | 42 |

## Instructional Results

Daily results. Students took a daily one-minute timed test from the Rocket Math program. The test consisted of 40 problems. All the students began the intervention on Set A, the first set in the program. In order to progress to the next set, a student had to achieve enough correct responses to meet his/her writing speed goal. For students whose writing speed was 40 , thirty-eight correct responses were required to pass to the next set.

Student A. Student A averaged 1.29 attempts in order to pass to the next fact set. He progressed more rapidly than any of the other students. His attempts ranged from one to three. Student A passed 20 of 23 levels in the Rocket Math multiplication program during the intervention.

Student B. Student B averaged 2.23 attempts in order to pass to the next fact set. Her attempts ranged from one to four. Student B passed 12 of 23 levels in the Rocket Math
multiplication program during the intervention.
Student C. Student C averaged 2.46 attempts in order to pass to the next fact set. Her attempts ranged from one to seven. Student C passed 12 of 23 levels in the Rocket Math multiplication program during the intervention.

Student D. Student D averaged 2.8 attempts in order to pass to the next fact set. Her attempts ranged from one to eight. Student D passed 10 of 23 levels in the Rocket Math multiplication program during the intervention.

Student $\boldsymbol{E}$. Student E averaged 2.0 attempts in order to pass to the next fact set. His attempts ranged from one to four. Student E passed 12 of 23 levels in the Rocket Math multiplication program during the intervention.

Student F. Student F averaged 2.18 attempts in order to pass to the next fact set. Her attempts ranged from one to five. Student F passed 10 of 23 levels in the Rocket Math multiplication program during the intervention.

Student $\boldsymbol{G}$. Student G averaged 1.77 attempts in order to pass to the next fact set. Her attempts ranged from one to four. Student G passed 13 of 23 levels in the Rocket Math multiplication program during the intervention.

Biweekly results. Students were given the Two-Minute Timed Multiplication Test biweekly. The test contained 100 basic multiplication facts with one-digit factors and products ranging from zero to eighty-one. The biweekly scores for each student are displayed in Figure 2.

Figure 2. Biweekly Two-Minute Timed Multiplication Test Scores


Student A. Student A's test scores on the Two-Minute Timed Multiplication Test were as follows: Pre-test: 26; Week 2: 60; Week 4: 65; and Week 6: 71. This represents a $131 \%$ increase from the baseline pre-test score to Week 2 . The increase from baseline to Week 4 was $150 \%$. The increase from baseline to Week 6 was $173 \%$.

Student B. Student B's test scores on the Two-Minute Timed Multiplication Test were as follows: Pre-test: 32; Week 2: 40; Week 4: 46; and Week 6: 62. This represents a $25 \%$ increase from the baseline pre-test score to Week 2. The increase from baseline to Week 4 was $44 \%$. The increase from baseline to Week 6 was $94 \%$.

Student C. Student C's test scores on the Two-Minute Timed Multiplication Test were as follows: Pre-test: 18; Week 2: 36; Week 4: 49; and Week 6: 47. This represents a $100 \%$ increase from the baseline pre-test score to Week 2 . The increase from baseline to Week 4 was $172 \%$. The increase from baseline to Week 6 was $161 \%$.

Student D. Student D's test scores on the Two-Minute Timed Multiplication Test were as follows: Pre-test: 14; Week 2: 36; Week 4: 40; and Week 6: 45. This represents a $157 \%$ increase
from the baseline pre-test score to Week 2 . The increase from baseline to Week 4 was $186 \%$. The increase from baseline to Week 6 was $221 \%$.

Student E. Student E's test scores on the Two-Minute Timed Multiplication Test were as follows: Pre-test: 25; Week 2: 31; Week 4: 40; and Week 6: 43. This represents a $24 \%$ increase from the baseline pre-test score to Week 2. The increase from baseline to Week 4 was $60 \%$. The increase from baseline to Week 6 was $72 \%$.

Student F. Student F's test scores on the Two-Minute Timed Multiplication Test were as follows: Pre-test: 25; Week 2: 36; Week 4: 41; and Week 6: 39. This represents a $44 \%$ increase from the baseline pre-test score to Week 2. The increase from baseline to Week 4 was $64 \%$. The increase from baseline to Week 6 was $56 \%$.

Student G. Student G’s test scores on the Two-Minute Timed Multiplication Test were as follows: Pre-test: 8; Week 2: 38; Week 4: 38; and Week 6: 49. This represents a $375 \%$ increase from the baseline pre-test score to Week 2 . The increase from baseline to Week 4 was $375 \%$. The increase from baseline to Week 6 was $513 \%$.

## Post-test Results

Post-test assessments included the Two-Minute Timed Multiplication Test and the Basic Multiplication Flash Card Assessment.

Two-minute timed multiplication test. The Two-Minute Timed Multiplication Test described in the Pre-test section was used as a bi-weekly test and as the post-test. The test contained 100 problems with one-digit factors and products ranging from zero to eighty-one. Table 6 contains the students' post-test scores for the Two-Minute Timed Multiplication Test (TMTM). The average number of seconds per problem (ANSM) on the Two-Minute Timed Multiplication Test was calculated for each student and is found in Table 6.

Table 6
Two-Minute Timed Multiplication Post-test Scores

| Student | TMTM | ANSM |
| :---: | :---: | :---: |
| A | $75 / 100$ | 1.6 |
| B | $63 / 100$ | 1.9 |
| C | $52 / 100$ | 2.3 |
| D | $41 / 100$ | 2.9 |
| E | $36 / 100$ | 3.3 |
| F | $38 / 100$ | 3.2 |
| G | $43 / 100$ | 2.8 |

[^0]Each participant made significant gains from the pre-test to the post-test scores. The scores showed gains ranging from $44 \%$ to $438 \%$. Each participant also showed a decrease in the average number of seconds required per problem on the test. These decreases range from $16 \%$ to $78 \%$, indicating increased automaticity for every student. The pre- and post-test scores for the Two-Minute Timed Multiplication Test are contained in Figure 3.

Figure 3. Pre- and Post- Two-Minute Timed Multiplication Test Scores


Student A. Student A correctly answered 75 of 100 problems as compared to his pre-test score of 26 , showing a $188 \%$ gain. He responded to the post-test problems with $100 \%$ accuracy and demonstrated understanding of the following facts: $0 \mathrm{~s}, 1 \mathrm{~s}, 2 \mathrm{~s}, 3 \mathrm{~s}, 4 \mathrm{~s}, 5 \mathrm{~s}, 6 \mathrm{~s}, 7 \mathrm{~s}, 8 \mathrm{~s}$, and 9 s . Student A averaged 1.6 seconds per problem on the post-test as compared to 4.6 on the pre-test. This showed a decrease of $65 \%$ in seconds required per problem indicating greatly improved automaticity.

Student B. Student B correctly answered 63 of 100 problems as compared to her pre-test score of 32 , showing a $97 \%$ gain. She responded to the post-test problems with $98 \%$ accuracy and demonstrated understanding of the following facts: $0 \mathrm{~s}, 1 \mathrm{~s}, 2 \mathrm{~s}, 3 \mathrm{~s}, 4 \mathrm{~s}, 5 \mathrm{~s}, 6 \mathrm{~s}, 7 \mathrm{~s}$, and 9 s . Student B averaged 1.9 seconds per problem on the post-test as compared to 3.8 on the pre-test. This showed a decrease of $50 \%$ in seconds required per problem indicating greatly improved automaticity.

Student C. Student C correctly answered 52 of 100 problems as compared to her pre-test score of 18 , showing a $189 \%$ gain. She responded to the post-test problems with $100 \%$ accuracy
and demonstrated understanding of the following facts: $0 \mathrm{~s}, 1 \mathrm{~s}, 2 \mathrm{~s}, 3 \mathrm{~s}, 4 \mathrm{~s}, 5 \mathrm{~s}$, and 9 s . Student C averaged 2.3 seconds per problem on the post-test as compared to 6.7 on the pre-test. This showed a decrease of $66 \%$ in seconds required per problem indicating greatly improved automaticity.

Student D. Student D correctly answered 41 of 100 problems as compared to her pre-test score of 14 , showing a $192 \%$ gain. She responded to the post-test problems with $100 \%$ accuracy and demonstrated understanding of the following facts: $0 \mathrm{~s}, 1 \mathrm{~s}, 2 \mathrm{~s}, 3 \mathrm{~s}$, and 5 s . Student D averaged 2.9 seconds per problem on the post-test as compared to 8.6 on the pre-test. This showed a decrease of $66 \%$ in seconds required per problem indicating greatly improved automaticity.

Student E. Student E correctly answered 36 of 100 problems as compared to his pre-test score of 25 , showing a $44 \%$ gain. He responded to the post-test problems with $100 \%$ accuracy and demonstrated understanding of the following facts: $0 \mathrm{~s}, 1 \mathrm{~s}, 2 \mathrm{~s}, 3 \mathrm{~s}, 4 \mathrm{~s}$, and 5 s . Student E averaged 3.3 seconds per problem on the post-test as compared to 4.8 on the pre-test. This showed a decrease of $31 \%$ in seconds required per problem indicating improved automaticity.

Student F. Student F correctly answered 38 of 100 problems as compared to her pre-test score of 25 , showing a $52 \%$ gain. She responded to the post-test problems with $100 \%$ accuracy and demonstrated understanding of the following facts: $0 \mathrm{~s}, 1 \mathrm{~s}, 2 \mathrm{~s}, 3 \mathrm{~s}, 4 \mathrm{~s}$, and 5 s . Student F averaged 3.2 seconds per problem on the post-test, as compared to 3.8 on the pre-test. This showed a decrease of $16 \%$ in seconds required per problem indicating improved automaticity.

Student G. Student G correctly answered 43 of 100 problems as compared to her pre-test score of 8 , showing a $438 \%$ gain. She responded to the post-test problems with $100 \%$ accuracy and demonstrated understanding of the following facts: $0 \mathrm{~s}, 1 \mathrm{~s}, 2 \mathrm{~s}, 3 \mathrm{~s}, 4 \mathrm{~s}, 5 \mathrm{~s}$, and 9 s . Student G averaged 2.8 seconds per problem on the post-test, as compared to 12.5 on the pre-test. This
showed a decrease of $78 \%$ in seconds required per problem indicating greatly improved automaticity.

## Basic multiplication flash card assessment. On the Basic Multiplication Flash Card

Assessment students were shown 100 flash cards containing single digit factors for the zero to nine facts to assess the students' automaticity with these basic facts. Students were asked to read the problem on the flash card and to give the answer. After reading the problem, if the student responded correctly within three seconds, the answer was considered correct. The scores for the Basic Multiplication Flash Card Assessment Post-Test scores are found in Table 7.

Table 7
Basic Multiplication Flash Card Post Test Scores

| Student | 0 s | 1 s | 2 s | 3 s | 4 s | 5 s | 6 s | 7 s | 8 s | 9 s |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | $10 / 10$ | $10 / 10$ | $10 / 10$ | $10 / 10$ | $\mathbf{7 / 1 0}$ | $10 / 10$ | $8 / 10$ | $9 / 10$ | $10 / 10$ | $9 / 10$ |
| B | $10 / 10$ | $10 / 10$ | $10 / 10$ | $9 / 10$ | $\mathbf{7 / 1 0}$ | $9 / 10$ | $8 / 10$ | $\mathbf{6 / 1 0}$ | $\mathbf{7 / 1 0}$ | $10 / 10$ |
| C | $10 / 10$ | $10 / 10$ | $10 / 10$ | $\mathbf{5 / 1 0}$ | $\mathbf{6 / 1 0}$ | $\mathbf{5 / 1 0}$ | $\mathbf{4 / 1 0}$ | $\mathbf{5 / 1 0}$ | $\mathbf{4 / 1 0}$ | $9 / 10$ |
| D | $10 / 10$ | $10 / 10$ | $10 / 10$ | $\mathbf{3 / 1 0}$ | $\mathbf{4 / 1 0}$ | $\mathbf{6 / 1 0}$ | $\mathbf{4 / 1 0}$ | $\mathbf{3 / 1 0}$ | $\mathbf{3 / 1 0}$ | $\mathbf{3 / 1 0}$ |
| E | $10 / 10$ | $10 / 10$ | $10 / 10$ | $8 / 10$ | $7 / 10$ | $10 / 10$ | $9 / 10$ | $\mathbf{5 / 1 0}$ | $8 / 10$ | $8 / 10$ |
| F | $10 / 10$ | $10 / 10$ | $10 / 10$ | $\mathbf{4 / 1 0}$ | $\mathbf{6 / 1 0}$ | $\mathbf{7 / 1 0}$ | $\mathbf{4 / 1 0}$ | $\mathbf{4 / 1 0}$ | $\mathbf{3 / 1 0}$ | $\mathbf{4 / 1 0}$ |
| G | $10 / 10$ | $10 / 10$ | $10 / 10$ | $9 / 10$ | $\mathbf{7 / 1 0}$ | $\mathbf{6 / 1 0}$ | $\mathbf{6 / 1 0}$ | $\mathbf{6 / 1 0}$ | $\mathbf{6 / 1 0}$ | $8 / 10$ |

Any answer requiring more than 3 seconds was considered incorrect. Bold type indicates facts not mastered with $80 \%$ or above automaticity. Grey highlight $=$ score increase from pre-test; Italics $=$ score decrease from pre-test.

All the participants increased their pre- to post test scores on the Basic Multiplication Flash Card Assessment. The students' gains ranged from an increase of 3\% to 93\%. Figure 4 shows the pre- and post test score information.

Figure 4. Basic Multiplication Flash Card Pre- and Post- Test Scores


Student A. Student A demonstrated automaticity by responding within three seconds to the following facts: $0 \mathrm{~s}, 1 \mathrm{~s}, 2 \mathrm{~s}, 3 \mathrm{~s}, 5 \mathrm{~s}$, and 8 s with $100 \%$ accuracy; 7 s and 9 s with $90 \%$ accuracy; 6s with $80 \%$ accuracy. Student A showed limited automaticity and lack of memorization as indicated by the following results: 4 s with $70 \%$ accuracy. He maintained $100 \%$ accuracy with the $0 \mathrm{~s}, 1 \mathrm{~s}$, and 5 s and made gains in the remaining fact families. He demonstrated improved automaticity by correctly answering 93 of 100 facts on the post-test compared to a pre-test score of 55 , showing a $69 \%$ gain.

Student B. Student B demonstrated automaticity by responding within three seconds to the following facts: $0 \mathrm{~s}, 1 \mathrm{~s}, 2 \mathrm{~s}$, and 9 s with $100 \%$ accuracy; 3 s and 5 s with $90 \%$ accuracy; and 6 s with $80 \%$ accuracy. Student B showed limited automaticity and lack of memorization as indicated by the following results: 4 s and 8 s with $70 \%$ accuracy; and 7 s with $60 \%$ accuracy. She demonstrated improved automaticity by correctly answering 86 of 100 facts on the post-test compared to a pre-test score of 69 , showing a $25 \%$ gain.

Student C. Student C demonstrated automaticity by responding within three seconds to the following facts: $0 \mathrm{~s}, 1 \mathrm{~s}$, and 2 s with $100 \%$ accuracy; 9 s with $90 \%$ accuracy. Student C showed
limited automaticity and lack of memorization as indicated by the following results: 4 s with $60 \%$ accuracy; $3 \mathrm{~s}, 5 \mathrm{~s}$, and 7 s with $50 \%$ accuracy; and 6 s and 8 s with $40 \%$ accuracy. Student C made gains in the following fact families: $1 \mathrm{~s}, 2 \mathrm{~s}, 6 \mathrm{~s}, 7 \mathrm{~s}, 8 \mathrm{~s}$, and 9 s ; stayed constant in the following: 0 s and 4 s ; and showed a decrease in the following: 3 s and 5 s . She demonstrated improved automaticity by correctly answering 68 of 100 facts on the post-test compared to a pre-test score of 60 , showing a $13 \%$ gain.

Student D. Student D demonstrated automaticity by responding within three seconds to the following facts: $0 \mathrm{~s}, 1 \mathrm{~s}$ and 2 s with $100 \%$ accuracy. Student D showed limited automaticity and lack of memorization as indicated by the following results: 5 s with $60 \%$ accuracy; 4 s , and 6 s with $40 \% ; 3 \mathrm{~s}, 7 \mathrm{~s}, 8 \mathrm{~s}$, and 9 s with $30 \%$ accuracy. Student D made gains in all of the fact families with the exception of the 9 s where she stayed constant. She demonstrated improved automaticity by correctly answering 56 of 100 facts on the post-test compared to a pre-test score of 29 , showing a 93\% gain.

Student E. Student E demonstrated automaticity by responding within three seconds to the following facts: $0 \mathrm{~s}, 1 \mathrm{~s}, 2 \mathrm{~s}$, and 5 s with $100 \%$ accuracy; 6 s with $90 \%$ accuracy; and $3 \mathrm{~s}, 8 \mathrm{~s}$, and 9s with $80 \%$ accuracy. Student E showed limited automaticity and lack of memorization as indicated by the following results: 4 s with $70 \%$ accuracy; and 7 s with $50 \%$ accuracy. Student E made gains in the following fact families: $5 \mathrm{~s}, 6 \mathrm{~s}$, and 8 s ; stayed constant in the following: $0 \mathrm{~s}, 1 \mathrm{~s}$, 2 s , and 3 s ; and showed a decrease in the following: 4 s , 5 s , and 9 s . He demonstrated improved automaticity by correctly answering 85 of 100 facts on the post-test compared to a pre-test score of 82 , showing a $4 \%$ gain.

Student F. Student F demonstrated automaticity by responding within three seconds to the following facts: $0 \mathrm{~s}, 1 \mathrm{~s}$, and 2 s with $100 \%$ accuracy. Student F showed limited automaticity
and lack of memorization as indicated by the following results: 5 s with $70 \%$ accuracy; 4 s with $60 \%$ accuracy; $3 \mathrm{~s}, 6 \mathrm{~s}, 7 \mathrm{~s}$, and 9 s with $40 \%$ accuracy; and 8 s with $30 \%$ accuracy. Student F made gains in the following fact families: $2 \mathrm{~s}, 6 \mathrm{~s}, 7 \mathrm{~s}$, and 9 s ; stayed constant in the following: 4 s and 8s; and showed a decrease in the following: 3 s and 5 s . She demonstrated improved automaticity by correctly answering 62 of 100 facts on the post-test compared to a pre-test score of 60 , showing a 3\% gain.

Student G. Student G demonstrated automaticity by responding within three seconds to the following facts: $0 \mathrm{~s}, 1 \mathrm{~s}$ and 2 s with $100 \%$ accuracy; 3 s with $90 \%$ accuracy; and 9 s with $80 \%$ accuracy. Student G showed limited automaticity and lack of memorization as indicated by the following results: 4 s with $70 \%$ accuracy; and $5 \mathrm{~s}, 6 \mathrm{~s}, 7 \mathrm{~s}$, and 8 s with $60 \%$ accuracy. Student G made gains in each fact family with the exception of the 5 s , where she remained constant. She demonstrated improved automaticity by correctly answering 78 of 100 facts on the post-test compared to a pre-test score of 49 , showing a $59 \%$ increase.

## Summary of Results

The question guiding this research project was, "Would the participants be able to develop automaticity with basic multiplication facts as a result of the concurrent implementation of systematic explicit instruction, peer tutoring, and progress monitoring?" The students made significant improvement as demonstrated by the results of the Two-Minute Timed Multiplication Test with scores showing gains ranging from $44 \%$ to $438 \%$. Each participant also showed a decrease in the average number of seconds required per problem on the test with decreases ranging from $16 \%$ to $78 \%$, indicating increased automaticity. All the participants increased their pre- to post-test scores on the Basic Multiplication Flash Card Assessment with gains ranging from an increase of $3 \%$ to $93 \%$. Clearly, the concurrent implementation of systematic explicit
instruction, peer tutoring, and progress monitoring impacted the students' development of automaticity with basic multiplication facts significantly.

## Discussion

## Teaching Procedures

Systematic explicit instruction. The method of systematic explicit instruction previously described was an effective component of the intervention as demonstrated by student scores. A possible addition to this component would be a weekly flash card review of previously learned facts. It would also be useful to incorporate skip counting into this component. However, the time teachers can spend on math fact instruction is limited. Thus, any additions to the intervention would need to be rotated with other parts of the program.

Peer tutoring. During the first four weeks of the intervention, students used the top part of their worksheet/test for peer tutoring. In Week 5, instead of peer tutoring, students asked to write the answers to the practice problems. This was allowed for one week. For the remainder of the intervention, students wrote answers to twenty practice problems and did peer tutoring with the remaining twenty problems.

Progress monitoring. At the beginning of the intervention the Rocket Math Two-Minute Timed Multiplication Tests were not available. However, the Carnine Two-Minute Timed Multiplication Test was available and was used as the pre-test. In Week 2 the students were given a Rocket Math Two-Minute Timed Multiplication Test as the first biweekly test, and the scores went down drastically due to test differences. The Carnine test progressed from easy to more difficult problems, while the Rocket Math did not. It was decided that using the Carnine test made the results more comparable.

The bi-weekly tests were an effective way to measure the students' progress, especially as they were used in conjunction with individual graphs. The daily one-minute timed tests were also a beneficial means to measure student learning. Again, these tests were paired with a graph to show how students were progressing through the math sets, as well as the number of trials required to pass each set.

The Basic Multiplication Flash Card Assessment was a useful tool for testing automaticity. The test was time consuming as it was administered orally to individuals presenting a significant drawback. However, the data gained from the test was very useful as a comparison to the written tests. One student with processing difficulties scored much higher on this test.

An interesting observation was made during the Basic Multiplication Flash Card Assessment post-testing. Three students, when shown the 9s flash cards were unable to give the answers, yet they were able to write these answers on their daily worksheets/tests. This raises the question of whether the skills gained in daily work transferred to other situations. A possible solution would be to incorporate a flash card drill into the peer-tutoring component once or twice per week.

## Challenges

Trying to complete the daily intervention components in ten minutes was the greatest challenge. During the first two weeks of the intervention, students missed class for multiple reasons. Getting the students to arrive on time was an initial challenge, but was solved with the Bee on Time Chart mentioned in the Incentives section.

There was concern for Student E, who has processing issues as a result of his learning disability. He knew more math facts on the Basic Multiplication Flash Card Assessment than his classmates, but was progressing through the sets more slowly than expected. Several solutions
were tried, i.e., oral testing and extending the timed test to two minutes. Student F had difficulty passing the tests, yet seemed to know all the answers. She demonstrated perfectionist tendencies, which slowed her writing process. She was coached to not erase and to allow herself to write messily. Her daily test time was increased to two minutes.

Another challenge was the amount of days that students were absent. Students missed an average of 5.2 days out of the 40 days of the intervention. Reasons for absences included sickness, school performances, and field trips. During the last month of the intervention, there were seven No School days verses thirteen days of school. Students received an entire week off for the Thanksgiving holiday, which preceded the week of post testing. To compensate for the week off, a letter and multiplication worksheets were sent to the parents encouraging them to practice with their children during the break. The students were offered an extra incentive for practice and were told they would earn ten bonus points toward a prize if they passed their oneminute test on the first day back to school. Three of the seven students won the incentive points by passing.

## Supplemental Outcomes

In addition to marked improvements made in the students' mastery and automaticity of the basic multiplication facts, other important life skill experiences were a natural outcome of the intervention. Students were taught the importance of getting to class on time and were required to sign-in on the Bee On Time Chart. The peer tutoring experience gave students opportunities to develop responsibility. Students were also in charge of retrieving their worksheet for the following day from the file box and placing it in their folder.

Character building was another supplemental outcome to the intervention. Student F was once offered ten extra seconds to finish a one-minute timed test, and she refused the offer saying,
"I can do it." Student E was allowed to test orally if he was having difficulty passing a set, because his processing issues made the written test a challenge. In the last week of the intervention, he was offered to test orally on his fourth trial of a set. He refused the offer saying that he could do it, and he was successful in passing. Even though, the students sometimes were disappointed in not passing a set, they developed confidence and perseverance.

## Limitations

Several limitations of this study should be noted. First, without a control group, it was difficult to attribute the students' score changes solely to the intervention. The eight-week duration of the project was also a limitation. Future studies should be conducted for a longer time period. This study focused on gaining automaticity in multiplication facts. However, future studies should incorporate the other math facts, as well as a wider age group.

## Acknowledgments

I would like to thank and acknowledge my mentor professor at Western Oregon University, Dr. Mary Scarlato, who has offered great insight and encouragement in this project. A special thanks is due to Dr. Mickey Pardew for providing the initial district contacts. In order to maintain the confidentiality of my students, I will use titles rather than names to thank those in the school district who helped me. A special thanks to the Coordinator of Special Programs, the school principal, my mentor teacher, and the special education assistant. Finally, I'd like to thank my husband for his encouragement and his willingness to proof read my report numerous times.

## References

Baker, S., Gersten, R., \& Lee, D. (2002). A synthesis of empirical research on teaching mathematics to low-achieving students. The Elementary School Journal, 103, 51-73.

Dixon, R. C., Carnine, D. W., Lee, D., Wallin, J., \& Chard, D. (1998). Report to the California state board of education and addendum to principal report review of high quality experimental mathematics research.

Fuchs, L. S., Fuchs, D., Powell, S. R., Seethaler, P. M., Cirino, J. M., \& Fletcher, J. M. (2008). Intensive intervention for students with mathematics disabilities: Seven principles of effective practice. Learning Disability Quarterly, 31(2), 79-92.

Kilpatrick, J., Swafford, J., \& Findell, B. (2001). Adding it up: Helping children learn mathematics. National Research Council.

Kroesbergen, E. \& Van Luit, J. (2003). Mathematics interventions for children with special educational needs. A meta-analysis. Remedial and Special Education, 24(2), 112.

National Mathematics Advisory Panel. (2008). Foundations for success: the final report of the national mathematics advisory panel. U.S. Department of Education: Washington, DC.

Rocket Math (2012). About Rocket Math. http://www.rocketmath.com/
Stein, M., Kinder, D., Silbert, J., \& Carnine, D. W. (2006). Designing effective mathematics instruction. A direct instruction approach. Upper Saddle River, New Jersey: Pearson.

Woodward, J. (2006). Developing automaticity in multiplication facts: Integrating strategy instruction with timed practice drills. Learning Disability Quarterly, 29(4), 269-289

## Appendix A

## Test Instruments Used

Untimed Addition/Subtraction Quiz
Two-Minute Timed Addition Test

Two-Minute Timed Subtraction Test
Two-Minute Timed Multiplication Test
Rocket Math Writing Speed Assessment
Multiplication Checklist

## Addition/Subtraction Quiz

Name
Class



| Pretest |  |  |  |  | Name Date $\qquad$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | 10 | 9 | 7 | 4 | 5 | 8 | 10 | 1 | 6 |
| -1 | -1 | -2 | -1 | -2 | -1 | -2 | -2 | -0 | -1 |
| 5 | 2 | 4 | 9 | 4 | 6 | 11 | 8 | 3 | 7 |
| -2 | -0 | -1 | -1 | -0 | -2 | -2 | -1 | -0 | -2 |
| 1 | 6 | 6 | 5 | 8 | 8 | 5 | 2 | 10 | 9 |
| -1 | -0 | -3 | -5 | -0 | -4 | -0 | -2 | -5 | -0 |
| 3 | 9 | 5 | 7 | 12 | 10 | 6 | 4 | 7 | 6 |
| -3 | -3 | -3 | -0 | -6 | -3 | -3 | -4 | -3 | -6 |
| 11 | 8 | 4 | 8 | 5 | 7 | 12 | 4 | 9 | 3 |
| -3 | -8 | -2 | -3 | -4 | $\underline{-7}$ | -3 | -3 | -9 | -2 |
| 6 | 8 | 5 | 8 | 7 | 7 | 6 | 9 | 9 | 10 |
| -5 | -7 | -3 | -6 | -6 | -5 | -4 | -8 | -7 | -9 |
| 6 | 10 | 12 | 7 | 10 | 11 | 14 | 8 | 11 | 9 |
| -3 | -8 | -9 | -4 | $\underline{-7}$ | -9 | -7 | -5 | -8 | -6 |
| 8 | 12 | 16 | 9 | 10 | 13 | 11 | 18 | 9 | 14 |
| -4 | -6 | -8 | -5 | -4 | -6 | -4 | -9 | -4 | -6 |
| 15 | 13 | 12 | 10 | 14 | 15 | 11 | 14 | 10 | 13 |
| -6 | -7 | -4 | -6 | -7 | -9 | -7 | -8 | -4 | -4 |
| 16 | 17 | 12 | 11 | 11 | 17 | 13 | 15 | 12 | 14 |
| -7 | -9 | -5 | -4 | -5 | -8 | -5 | -7 | -8 | -5 |

Pretest
Name
Date $\qquad$

| 1 | 4 | 5 | 1 | 2 | 5 | 3 | 3 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $\times 1$ | $\times 5$ | $\times 2$ | $\times 5$ | $\times 2$ | $\times 2$ | $\times 4$ | $\times 1$ |
| $\times 5$ | $\times 2$ |  |  |  |  |  |  |

$\begin{array}{rrrrrrrrr}1 & 5 & 5 & 2 & 5 & 2 & 1 & 3 & 4 \\ \times 8 & \times 5 & \times 1 & \times 5 & \times 3 & \times 2 & \times 4 & \times 2 & \times 2\end{array}$

| 0 | 5 | 5 | 2 | 5 | 2 | 9 | 5 | 2 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $\times 1$ | $\times 6$ | $\times 9$ | $\times 0$ | $\times 8$ | $\times 8$ | $\times 0$ | $\times 7$ | $\times 7$ |

$\begin{array}{rrrrrrrr}0 & 2 & 6 & 8 & 0 & 6 & 2 & 7 \\ \times 6 & \times 9 & \times 5 & \times 2 & \times 2 & \times 6 & \times 5 & \times 2 \\ \times 2\end{array}$

| 7 | 9 | 0 | 9 | 9 | 2 | 0 | 9 | 4 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $\times 2$ | $\times 4$ | $\times 8$ | $\times 5$ | $\times 2$ | $\times 9$ | $\times 7$ | $\times 3$ | $\times 2$ |


| 4 | 3 | 0 | 9 | 5 | 0 | 4 | 4 | 4 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $\times 4$ | $\times 9$ | $\times 6$ | $\times 2$ | $\times 4$ | $\times 9$ | $\times 3$ | $\times 9$ | $\times 4$ |

$\begin{array}{rrrrrrrrr}9 & 6 & 4 & 9 & 4 & 2 & 9 & 9 & 9 \\ \times 8 & \times 9 & \times 6 & \times 9 & \times 8 & \times 4 & \times 6 & \times 4 & \times 9 \\ \times 7\end{array}$

| 3 | 6 | 7 | 4 | 7 | 3 | 8 | 8 | 9 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $\times 9$ | $\times 4$ | $\times 9$ | $\times 9$ | $\times 4$ | $\times 4$ | $\times 9$ | $\times 4$ | $\times 7$ |


| 7 | 7 | 6 | 3 | 8 | 6 | 3 | 9 | 6 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $\times 6$ | $\times 3$ | $\times 6$ | $\times 6$ | $\times 3$ | $\times 8$ | $\times 7$ | $\times 3$ | $\times 7$ |


| 9 | 7 | 8 | 3 | 8 | 9 | 8 | 9 | 6 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $\times 8$ | $\times 7$ | $\times 6$ | $\times 3$ | $\times 8$ | $\times 7$ | $\times 6$ | $\times 8$ | $\times 7$ |

## Mastering Math Facts

mxx
Name $\qquad$
Assessments

## How fast can you write?

Wait for my signal to begin. You will have I minute to copy the numbers shown in the comer of each box. Write as quickly as you can. Ready, set, go!


7 bazes


14 baxes


35 baxes


Count how many boxes you completed. $\qquad$


[^0]:    Note: TMTM = Two-Minute Timed
    Multiplication Test; Average Number of Seconds Per Problem. Test administered November 26-29, 2012.

