

The Pennsylvania State University

The Graduate School

College of Education

**EFFECTIVENESS OF MASTERING MATH FACTS ON SECOND- AND THIRD-
GRADE STUDENTS WITH SPECIFIC LEARNING DISABILITIES IN
MATHEMATICS**

A Dissertation in

Special Education

by

Karen M. Fries

© 2013 Karen M. Fries

Submitted in Partial Fulfillment
of the Requirements
for the Degree of

Doctor of Philosophy

August 2013

The dissertation of Karen M. Fries was reviewed and approved* by the following:

Paul J. Riccomini
Associate Professor of Special Education
Dissertation Adviser
Chair of Committee

Richard M. Kubina
Associate Professor of Special Education, Professor In Charge of Special Education Program

David L. Lee
Associate Professor of Special Education

Gwendolyn M. Lloyd
Professor of Education (Mathematics Education)

*Signatures are on file in the Graduate School.

ABSTRACT

Students with specific learning disabilities (SLD) in mathematics can typically experience long-lasting problems when developing fluency with basic math facts despite the use of evidence-based practices. One recently developed intervention to promote memorization and fact-fluency is Mastering Math Facts (MMF). MMF is a commercially-available program that is in use in schools across the United States, yet there is currently limited empirical evidence that shows its impact as an intervention for students with SLD. The current study investigated the effectiveness of MMF for four second- and third-grade students with SLD in mathematics by measuring digits correct per minute (dc/m) and per 2 minutes (dc/2m) on daily curriculum-based measures and weekly 2-minute math facts timings, respectively. Improvements were noted across all students' dc/m and dc/2m scores from their pre-test and baseline measures following implementation of MMF. Implications for educators are provided.

Keywords: basic math facts, math facts fluency, mastering math facts, elementary students with learning disabilities in mathematics

TABLE OF CONTENTS

List of Tables.....	v
List of Figures.....	vi
Chapter 1. INTRODUCTION.....	1
Chapter 2. METHODS.....	9
Chapter 3. RESULTS.....	21
Chapter 4. DISCUSSION.....	34
References.....	43
Appendices	
Appendix A: Tables.....	49
Appendix B: Figures.....	53
Appendix C: Review of Relevant Research.....	58

LIST OF TABLES

Table 1: Participant Characteristics.....	49
Table 2: Empirically Derived Fluency Criteria for Second- and Third-Grade Students... 16	
Table 3: Single-Skill Digits Correct and Errors per Minute: Means and Standard Deviations.....	50
Table 4: Single-Skill Digits Correct and Errors per 2-Minutes: Means and Standard Deviations.....	51
Table 5: Mixed-Skills Digits Correct and Errors per Minute: Means and Standard Deviations.....	52

LIST OF FIGURES

Figure 1: Fluency Standards for Grades 2 and 3..... 1

Figure 2: Daily 1-Minute Single-Skill CBMs Across Baseline, Intervention, and Maintenance Phases..... 53

Figure 3: Weekly 2-Minute Single-Skill CBMs Across Baseline, Intervention, and Maintenance Phases..... 54

Figure 4: Weekly 1-Minute Mixed-Skills CBMs Across Baseline, Intervention, and Maintenance Phases..... 55

Figure 5: Sample Mastering Math Facts lettered set..... 56

Figure 6: Sample Mastering Math Facts Rocket Chart..... 57

Chapter 1

Introduction

Between 5% and 8% of school-age children experience some form of specific learning disabilities (SLD) in mathematics that interferes with their ability to learn mathematical concepts or procedures (Geary, 2004). Additionally, many students with SLD may have difficulties with retrieval of basic facts from long-term memory, which is of significant concern for educators. Since early performance deficits are closely linked with long-term failure in mathematics (Griffin & Case, 1997; Morgan, Farkas, & Wu, 2011), children who have not acquired critical prerequisite skills, such as fluency with basic facts, often lag behind their peers in academic achievement throughout their school careers as the performance gap continues to widen (McClelland, Acock, & Morrison, 2006; National Mathematics Advisory Panel, NMAP, 2008). By the end of Grade 5, students should be proficient with addition, subtraction, multiplication, and division of whole numbers (NMAP, 2008); therefore, the teaching of basic mathematics skills such as memorization of math facts is a critical component of elementary school years.

Furthermore, the Common Core State Standards for Mathematics expects proficiency with multi-digit multiplication (as well as addition and subtraction) problems as early as the end of Grade 4, with proficiency for certain single-digit skills expected as early as the end of Grade 2 (Common Core State Standards Initiative, CCSSI, 2010). Figure 1 illustrates specific fluency standards for both Grades 2 and 3.

Figure 1
Fluency Standards for Grades 2 and 3

<p>Grade 2, Operations and Algebraic Thinking Standard 2.OA.2: Fluently add and subtract within 20 using mental strategies. By the end of Grade 2, know from memory all sums of two one-digit numbers (CCSSI, 2010, p. 19).</p>	<p>Grade 3, Operations and Algebraic Thinking Standard 3.OA.7: Fluently multiply and divide within 100. By the end of Grade 3, know from memory all products of two one-digit numbers (CCSSI, 2010, p. 23).</p>
--	--

As Stein, Silbert, and Carnine (2006) discuss, there are 390 basic math facts students are expected to learn, which poses a significant challenge to ensure all students are proficient by the end of Grade 3.

Mathematic difficulties, however, are not just a school-related concern. Research has repeatedly demonstrated mathematics competence is important for success not only in school, but also impacts future employment, income levels, and work productivity (i.e., Flores, 2009; Rivera-Batiz, 1992). As children progress through school, mathematics competency is critical not just for academic performance, but also for daily living skills such as earning an income, budgeting, and knowing appropriate ways to spend money (Cihak & Grim, 2008). Almost 60 percent of American adults have mathematics skills below levels considered necessary for meeting demands of everyday life and work, such as planning and following schedules, completing routine tasks on the computer, banking, and solving problems (Vukovic & Siegel, 2010).

Vukovic and Siegel (2010) confirm mathematics difficulties in early years, including lack of automaticity with basic math facts, can have serious consequences for job performance, employability, and daily living, not to mention more complex skills in mathematics. If students with SLD continue to use inefficient counting strategies, such as counting on their fingers or drawing tally marks, they will be unable to understand more complex skills like multi-digit computation problems, fractions, or algebra (Gersten & Chard, 1999). Given that students with SLD in mathematics often demonstrate persistent difficulties in memory retrieval, learning and applying mathematical skills, and strategic processes that limit their ability to solve basic facts and whole-number computation problems (Bryant, Hartman, & Kim, 2003), effective instruction in all mathematical areas, including basic math facts, is especially critical for these students.

Effective Mathematics Instruction

A meta-analysis conducted by Baker, Gersten, and Lee (2002) found support for explicit instruction, including the teaching of rules, principles, basic math facts, and problem-solving strategies based on Engelmann and Carnine's (1982) direct instruction techniques. Baker et al. noted the importance of extensive teacher modeling of components in mathematics lessons along with close supervision, monitoring, and immediate feedback as methods to ensure student success. Baker et al. discussed the importance of including significant amounts of carefully constructed practice as a successful instructional technique for teaching students of all ability levels mathematics operations, concepts, and procedures.

Fuchs et al. (2008) identified seven principles of effective instruction and intervention for students with SLD in mathematics including: instructional explicitness, thoughtful instructional design, drill and practice incorporating activities such as flashcards and daily practice, cumulative review over time, and continuing progress monitoring. Similarly, Smith and Geller (2004) found effective procedures and cognitive strategies to reach all students include: teacher modeling where the teacher presents both concepts and necessary steps to solve problems, guided practice where the teacher monitors student practice before allowing for independent practice, feedback where the teacher provides systematic, immediate corrective feedback to students, and review opportunities that occur over time to help students reach fluency and generalization goals. Swanson (1999) noted direct instruction techniques including teacher modeling, the use of small interactive groups, drill-repetition, and practice-review incorporating daily and/or weekly review activities were beneficial in helping to remediate academic difficulties for students with SLD in mathematics.

Basic Math Facts

An area of noteworthy concern is the ability of students with SLD to automatically and fluently compute basic math facts. Fluency (e.g., number of correct digits per minute) is often used to illustrate both accurate and rapid responses to a whole group of problems, such as addition or subtraction problems (Miller, Skinner, Gibby, Gaylon, & Meadows-Allen, 2011). In order for students to achieve fluency in computation, they should have the ability to compute basic math facts and directly retrieve said facts without exerting unnecessary amounts of cognitive energy (Woodward, 2006). However, children with SLD often have difficulty in the area of automaticity, lack numerical concepts, and do not have basic facts memorized (Stock, Desoete, & Roeyers, 2010). Research demonstrates that difficulty with retrieval skills is the most commonly occurring computational deficit for children with SLD; this deficit is, in fact, one of the defining aspects of students identified with SLD (Jordan, Kaplan, & Hanich, 2002).

Hasselbring, Goin, and Bransford (1988) found that by age 12, students without disabilities are able to recall three times as many basic math facts as compared to students with SLD. Given there are 390 basic facts with which students are expected to be proficient by the end of third grade (Stein et al., 2006), early instruction and intervention with basic facts is crucial. The NMAP (2008) determined that computational proficiency with whole number operations is not only a critical skill for children to master but that it is dependent on significant amounts of practice to develop automatic recall of and fluency with addition and multiplication facts. A recent IES guide (Gersten et al., 2009) similarly identified the importance and statistically significant positive effects of including activities to practice fact fluency as a necessary component for students who struggle in or may need assistance with mathematics to ensure long-term success in mathematics for these struggling students. Gersten et al. found

supportive evidence for systematic focused interventions with counting strategies and properties of operations, especially with younger students, as beneficial to promoting long-term growth in mathematical areas other than just basic fact retrieval. In fact, Gersten and colleagues recommend that instruction at all grade levels regardless of disability status of the students, should spend about 10 minutes of every intervention session working to build fluent retrieval of those 390 basic facts.

General education students and instruction of basic facts. The National Council of Teachers of Mathematics (NCTM, 2006) determined essential areas of competency with whole number arithmetic include knowledge of basic mathematics facts, skills in using standard algorithms for solving more complex problems, estimating answers, and understanding of core mathematical concepts. A literature search of existing research about math facts instruction for students without disabilities found that students typically progress from a “counting all” strategy (e.g., count all addends separately then counting them together) to a “counting on” strategy (e.g., count on after saying the first addend) to a “counting-on-to-the-larger-addend” strategy (e.g., selecting the larger addend and counting on from there) where “1” is not always the first addend (Ashcraft & Christy, 1995). Most students without learning disabilities are able to progress at least as far as the “counting on” strategy to become more fluent and automatic when solving basic math facts (Garnett, 1992). While these strategies are beneficial for students first learning basic math facts, continued use of such strategies is inefficient, will prevent students from becoming fluent with basic facts, and will limit the amount of energy students have to understand more complex skills such as multi-digit computation problems, fractions, or algebra (Gersten & Chard, 1999).

Educators often use concrete objects or hands-on activities using manipulative objects (Witzel, Riccomini, & Schneider, 2008) to assist students in deriving accurate answers to basic math facts problems, which again may be especially useful during initial learning of such basic facts. Students without disabilities are often able to move beyond the use of manipulatives and learn to solve basic math facts automatically and without thinking about the problems (Pellegrino & Goldman, 1987). Although the use of manipulatives, number lines, or other teaching strategies may be efficient procedures to enhance accuracy, conceptual understanding of targeted skills, and conceptual understanding of related concepts, students may become dependent upon those concrete objects or other materials which may end up taking up a significant amount of time that could instead be used to solve more complex mathematical problems (Poncy, Skinner, & Jaspers, 2007). With regard to solving of basic facts and in contrast to peers in other countries, students in the United States have not always memorized all of the basic facts, do not always reach a level of expected automaticity, and must continue to use inefficient strategies such as “count all” or concrete manipulatives in order to solve such problems (e.g., Fuson, 1982; Geary, Bow-Thomas, Liu, & Siegler, 1996; Geary et al., 2008).

Many students without learning disabilities progress from trying to figure out basic math facts utilizing a variety of strategies to memorizing the answers and just writing them down. Haring and Eaton (1978) suggest learning proceeds through three stages starting with the acquisition stage, then the skill fluency stage, and concluding with the generalization stage; Lin and Kubina (2005) note that improvements with fact fluency in the early stages of learning can lead to increases in later stages of learning. Students without learning disabilities are often more apt to progress to the generalization stage, while students with SLD in mathematics often remain

confused or stuck at the acquisition or fluency stages of learning (Poncy, Duhon, Lee, & Key, 2010).

Instruction in basic facts for students with disabilities. One necessary, but often lacking instructional component specifically beneficial to students with disabilities is repeated practice. Repeated practice has been found to promote accurate responding to math problems and to assist students with moving beyond the acquisition stage (Poncy et al., 2010). Although Kroesbergen and Van Luit's (2003) meta-analysis of mathematics interventions for students with special needs identified aforementioned direct instruction methods as the most effective instructional strategies for students to learn basic math facts, recent reviews in the area of mathematics point out that traditional math programs often fail to provide direct and explicit demonstrations of key skills, frequent opportunities for students to practice target skills, and procedures for providing corrective feedback to students (Doabler et al., 2012). Similarly, students are often expected to learn basic facts in groups of 10 or more, are expected to move on to new facts before mastery has occurred, and are often only provided "instruction" through mad minute assessments, which do not actually teach anything (Crawford, 2003). Without direct intervention in mathematics concepts and procedures including basic facts, learning disabilities in mathematics will most likely persist into adulthood (Miller & Mercer, 1997).

Research Questions

One commercially available program that provides such opportunities for explicit demonstrations, repeated practice, and academic feedback is Mastering Math Facts (Crawford, 2003). Mastering Math Facts (MMF) is currently in use in thousands of schools (www.oci-sems.com) across the United States, yet has not been included in any reviews that investigated necessary components of basic-fact fluency interventions. MMF includes a number of

components previously identified as effective practices to instruct students in basic math facts, such as multiple opportunities for practice with modeling, the use of flashcards, instruction in only small sets of facts at one time, and immediate performance feedback. Therefore, the purpose of this study was to evaluate the impact of the MMF intervention on math-fact fluency of students with disabilities in a northeastern elementary school. The following four experimental questions guided the study:

1. Can students with SLD in mathematics reach expected grade-level single-skill fluency goals using MMF as a supplemental intervention over an 8-week intervention period?
2. Does the mixed-skills performance of students with SLD in mathematics improve as a result of MMF?
3. Is the use of MMF an efficient intervention for teachers to use with students with SLD in mathematics?
4. How socially valid and acceptable is MMF as intervention for students with SLD in mathematics?

Chapter 2

Methods

Participants and Setting

Four second- and third-grade students (3rd grade, N=3) served as participants in this study. All students received special education services for SLD in mathematics at the time of this study and had mathematics goals on their individualized education programs (IEP). All four students received their entire mathematics instruction in a learning support classroom separate from their typical second- or third-grade classrooms for 30-minutes daily. Direct instruction was provided to all four students using the Connecting Math Concepts curriculum. All four students were females, two were Caucasian (Cheryl and Amanda) and two were African-American (Marilyn and Alexa). The study took place in a university community in central Pennsylvania at an elementary school serving students in Kindergarten through 5th grade. Pre-test assessments were conducted one week prior to implementation of MMF; intervention began the first week of February and continued for 40 days. Intervention sessions occurred in a corner of the learning support classroom from which the participants were selected. Intervention occurred at the same time each day, Monday through Friday, at the specific time determined by the classroom teacher. Table 1 shows participant demographic information and scores from three pre-test measures.

Materials

Mastering Math Facts. Mastering Math Facts (MMF, Crawford, 2003) subtraction materials were used to implement the intervention. This program is a structured basic math facts intervention in which math facts are broken down into just two facts and their inverses (e.g., 5-1, 5-4) to be mastered at one time (Crawford, 2003). MMF is designed for use with elementary school students struggling to learn and become proficient with basic math facts, especially in

grades 2 -5 or with older students (e.g., middle school). MMF is structured the same way for all four operations and involves the use of lettered sheets A-Z for each operation. This intervention promotes the memorization of math facts in small sets (e.g., 3 - 1, 3 - 2, 4 - 1, 4 - 3 is a complete set) which is intended to allow students to memorize basic math facts more easily. MMF was not designed specifically for students with SLD, as it is in fact appropriate for students of all ability levels (Crawford, 2003). The researchers were not affiliated with the MMF program.

General implementation procedures. Instruction in MMF is presented visually and auditorily using flashcards and oral practice drills, and can be presented to a small group, individual students, or even a whole group if the whole group is learning the same basic math facts (set) at the same time. Each set A-Z only consists of 2 facts, their inverses, and perhaps a doubles fact (e.g., 10 - 5, 12 - 6, 14 - 7) to learn at once, except for two exceptions in the operation of multiplication. Regardless of operation, there are 40 practice problems on the top half of each lettered sheet and 40 timed assessment problems on the bottom of each lettered sheet. Both halves of the lettered sheets consist of new facts mixed in only with facts that have already been mastered; in the case of set A, only those facts (e.g., 3 - 1, 3 - 2, 4 - 1, 4 - 3) from set A comprise both the top and bottom halves. Figure 5 shows a sample lettered set in MMF.

Mastery of a lettered set as defined by Crawford (2003) is 38/40 problems in 1-minute, and students do not move onto the next letter until they master their current lettered set. However, handwriting issues are a separate concern and are taken into account if necessary. Regardless of handwriting concerns, students are given three minutes to practice on the top half of lettered sheets and then take a 1-minute test on the bottom half. Instruction is provided for new math facts (the top half of lettered sheets) by showing students flashcards, including the answer, for new and previously learned facts. For example, a card looks like “3 – 1 = 2” and the

whole problem is said along with the answer. Once a student masters a set, he or she moves on to the next letter. When a student does not pass a set on the first attempt, he or she has five more attempts to do so. If he or she still does not pass after six total attempts, he or she moves back up to the letter just prior to where he or she was struggling. If he or she does not pass that lettered set, more instruction or intervention is provided from the researcher or teacher on the set in which he or she was struggling.

Procedures

Pre-test measures. MMF was implemented by the primary researcher with four students, beginning one at a time to follow multiple-probe protocols, starting the first week of February. As a pre-test measure, the primary researcher administered a numeral copying assessment to ensure writing speed would not present as a limiting factor for participants on timed math-facts assessments. Students were administered the “How Fast Can You Write” writing assessment included with MMF in order to implement the MMF intervention with fidelity. In this assessment, students were asked to copy numerals from a corner of a box into the empty space of the box for one minute. Crawford (2003) suggests at least 25 digits should be correctly copied in that one minute before implementation of MMF.

All students in a learning support classroom were administered the numeral copying assessment along with five other pre-test measures. Students were given a packet composed of the numeral copying assessment, a 1-minute single-skill curriculum-based measure (CBM) retrieved from www.Aimsworld.com, a 2-minute single-skill timing included with MMF, and a 1-minute mixed-skills CBM retrieved from www.Aimsworld.com to determine who qualified for participation in the study. Per the teacher’s suggestion, students were assessed in both addition and subtraction to determine the most appropriate operation for the study. Standardized

directions were read to all students (from Hosp, Hosp, & Howell, 2007, p. 104). Students were instructed to skip problems they may not know how to solve and to mark an X through any problems where they felt they made a mistake as opposed to erasing the problem. Both 1-minute measures were scored as digits correct per minute (dc/m) and the 2-minute timing was scored as digits correct per 2-minutes (dc/2m). Digits were counted as correct if the answer was correct and each digit was in the correct place (Deno & Mirkin, 1977).

The four students who had both the lowest dc/m other than 0 dc/m on the pre-test single-skill measures and who had mathematics goals on their IEPs were selected for participation. Based on the Aimsweb growth table from 2006-2007, all four selected students had pre-test scores below the 10th percentile. Students in the classroom who scored 0 dc/m on pre-test measures were removed from consideration for participation, as the classroom teacher did not feel they were ready for work with basic facts since they were still learning to identify and write numerals, count concrete objects, and discriminate quantity when presented with 2 numbers. Pre-test measures were conducted one week prior to collection of baseline data; parental consent and student assent forms were sent home to qualifying participants. Beginning one week after completion of pre-test measures, the primary researcher administered daily baseline measures (1-minute single-skill CBMs) to all students who qualified for participation in a learning support classroom at the same time until at least 6 data points were collected. Weekly 2-minute single-skill MMF timings and 1-minute mixed-skills CBMs occurred once a week during both baseline and intervention phases. No changes in instruction took place during this baseline phase.

Intervention. For Student 1 and then all subsequent students, the primary researcher began each instructional session with a 1-minute single-skill CBM retrieved from Aimsweb.com to assess students' progress prior to daily instruction. Participants were also given both 2-minute

single-skill timings from MMF and 1-minute mixed-skills CBMs one time each week; 1-minute CBMs occurred on Monday before instruction began for the day and 2-minute MMF timings took place on Fridays after daily instruction. The primary researcher then introduced new facts for the appropriate lettered set or reviewed facts from the un-mastered set upon which the student was working, using both flashcards and oral practice. Students were allowed 3 minutes to orally practice the facts and then write answers on the top half of the worksheet for the given lettered set.

In the current study, flashcards were used to practice new facts and to review previously learned facts. The researcher used a small deck of 12 –15 cards at one time and usually only 2-5 of these cards were new depending on the number of new facts in each set. To practice, students said the whole problem and answer after the whole problem and answer were provided by the researcher. When the student made an error or a hesitation of 3 seconds was evidenced, the researcher gave the correct answer immediately. The researcher said, “Stop, you missed that one. 2 minus 1 equals 1. Say it with me. Now you say it. Good, what is 2 minus 1?” Students repeated the whole problem and answer with the researcher and then repeated the whole problem and answer individually. The researcher put any incorrect cards back into the deck 2 - 4 cards from the front so students had almost immediate re-exposure to missed facts. As unlearning incorrect basic math facts takes a great deal of time and cognitive effort, the researcher made certain students had multiple opportunities for feedback and were practicing facts correctly.

After practicing on the top half of the lettered sheets, feedback was provided by the primary researcher, and students were told it was time to take the test for the day in which they had 1-minute to complete as much of the bottom half of the worksheet as they could before time was called. When they completed the test for the day, the primary researcher scored the bottom

half of the lettered sheet and discussed their progress towards mastery (38/40 problems correct/minute per lettered set) of the set. When students passed a set, they were able to color in the corresponding section on their individual Rocket chart (see Figure 6), and were told they would move on to the next set the following day. When students did not pass, they were told how many more they needed to answer correctly the next day in order to pass to the next lettered set. Students who did not pass a set after six attempts moved back to the previous lettered set.

Design

To effectively answer the primary and the first secondary research questions: Can students with SLD in mathematics reach expected grade-level fluency goals using MMF as a supplemental intervention over an 8-week intervention period?, and, Does the mixed-skills performance of students with SLD in mathematics improve as a result of MMF?, a single-case multiple probe design was implemented. Each student received their mathematics instruction in the same learning support classroom from the same special education learning support teacher. Implementation of MMF occurred in the same learning support classroom as students' typical mathematics instruction.

All four students began baseline at the same time; baseline data were collected for a minimum of six days and until the first student reached stability. In order to determine if MMF led to behavioral change in terms of increases in students' dc/m or dc/2m, stability was defined as "a state in which the behavior in question does not change its characteristics over time" (Sidman, 1960, p. 234). Using guidelines suggested by Sidman, the mean of the first three days of data collection were compared with the mean of the last three days of data collection. Then, "if the difference between these means was less than" 50 percent "of the six days' mean," (p. 260), the participant was considered to have reached stabilization. When the first student

reached stability in the baseline phase, this student (Cheryl) began instruction in MMF while the other students continued to receive only typical classroom instruction and continued with multiple-probe data collection procedures. For the other three students, baseline data were collected every third day until Student 1 mastered three lettered sets of the intervention (e.g., Set C) and Student 2 demonstrated stability in the baseline phase, at which point Student 2 began intervention.

Baseline data were collected for students not yet in the intervention phase on days both immediately before and after any other student entered intervention. Data collection proceeded in the same way for all students still in baseline, while data were collected each day for students in intervention. Student 3 began intervention after demonstrating stability in baseline and after Student 2 passed Set C. Student 4 began intervention after demonstrating stability in baseline and after Student 3 passed Set C. During baseline, students continued to receive typical classroom mathematics instruction without any additional practice with math facts other than what was provided in the math curriculum. A multiple-probe design was selected as the most appropriate design for this study for two reasons: (1) the specific behavior of math-fact fluency could be identified and measured over time in order to show baselines against which changes in the behavior could be evaluated, and (2) this type of design helped to alleviate the problem of having to withdraw an intervention (which is difficult to achieve with an educational intervention to which students have already been exposed) to show control of the behavior by the intervention (Murphy & Bryan, 1980).

Dependent Variables

Because fluent computation of math facts is a critical mathematics skill for children to master and students with SLD often have difficulties with this skill, the researcher employed

three dependent measures to determine the effects of MMF for students with SLD in mathematics. Dependent measures consisted of: 1-minute single-skill CBMs, 2-minute single-skill timings from MMF, and 1-minute mixed-skills CBMs. To establish realistic fluency goals for second- and third-grade students with SLD, the primary researcher considered goals suggested in extant literature by a number of researchers. Table 2 shows a range of fluency rates for second- and third-grade students as suggested in current literature.

Table 2
Empirically Derived Fluency Criteria
For Second- and Third-Grade Students

Researchers or Assessment System	Digits Correct per Minute for Instructional Level
Aimsweb 2 nd grade, 75 th percentile (2008)	30
Aimsweb 3 rd grade, 75 th percentile (2008)	37
Burns, VanDerHeyden, Jiban (2006)	14-31
Deno and Mirkin (1977)	21-40
Lin and Kubina (2005)	60-90
Miller and Heward (1992)	30-40

Considering the rather wide spread of established fluency goals and the fact that participants in this study were all identified with SLD in mathematics, a fluency goal was set at 50 dc/m or greater for single-skill subtraction facts for second- and third-grade students.

One-minute single-skill CBMs. The first outcome measure was single-skill (subtraction facts) CBMs retrieved from www.Aimsweb.com and administered daily for one minute. The dependent variable (DV) was digits correct per minute (dc/m). Digits were counted as correct if the answer was correct and each digit was in the correct place (Deno & Mirkin, 1977). Students completed this CBM each day prior to both instruction in new math facts and/or a review of previously-learned facts. Although MMF includes 1-minute daily timings that consist only of facts from the current lettered set and facts from previously mastered sets for a total of 40

problems per 1-minute timing, no psychometric properties have been reported for the timings. The 1-minute timings included with MMF could inflate the total number of dc/m as again, these timings only cover facts students already learned and mastered. Therefore, 1-minute single-skill CBMs from Aimsweb were used as one of the dependent measures to demonstrate possible effectiveness of MMF (Aimsweb Mathematics Computation: Administration and Technical Manual, 2010). Data from daily 1-minute CBMs were graphed on an equal interval graph to visually analyze changes in level and trend after implementation of MMF.

Two-minute single-skill MMF timings. The second dependent measure was single-skill (subtraction facts) timings included with the MMF program and administered weekly for two minutes. The DV was digits correct per two minutes (dc/2m) using Deno and Mirkin's (1977) aforementioned criteria for counting digits as correct. Two-minute timings were administered once weekly, always on Friday and always after instruction for that day. Students were trained to count their own number of digits correct, the researcher verified their counts, and students graphed their progress on an equal-interval graph. This allowed students to individually observe their weekly progress and growth. MMF includes 10, 2-minute timings consisting of a total of 80 problems; 2-minute timings were selected randomly each week and the same timing was administered to all four students. The 2-minute timings consist of all basic facts for the specific operation rather than just facts students will already learned or mastered due to instruction with MMF.

One-minute mixed-skills CBMs. The third outcome measure was mixed-skills (addition and subtraction facts) CBMs retrieved from www.Aimsweb.com and administered weekly for one minute. The DV was dc/m. This measure was exactly the same as the single-skill CBMs previously discussed, except both addition and subtraction problems were included. Students

completed this CBM once weekly, always on Monday and always prior to both instruction in new math facts and/or a review of previously-learned facts. Data from weekly 1-minute mixed-skills CBMs were graphed on an equal interval graph to visually analyze changes in level and trend after implementation of MMF.

Maintenance. To assess maintenance effects of MMF, a 1-minute single-skill CBM from Aimsweb.com, a 2-minute single-skill weekly timing from MMF, and a 1-minute mixed-skills CBM from Aimsweb.com were administered weekly starting two weeks after completion of the study and continuing for a total of three weeks. Therefore, maintenance effects were measured two weeks, three weeks, and four weeks after the fortieth day of the study was completed.

Procedural Fidelity and Reliability

Procedural fidelity. To determine procedural fidelity, the researcher developed a checklist for use one time per week after a session of MMF takes place. Checklist items included, for example, Did the researcher implement all steps of MMF correctly? Were flashcards properly used to practice new math facts? The researcher enlisted the assistance of an independent, trained observer to observe 20% of all sessions and complete the procedural fidelity checklist.

As a second measure of procedural fidelity, the researcher ensured students only received instruction in math facts during intervention time and that business-as-usual instruction was what occurred during typical math instruction in the students' classroom. Students who received the intervention did not miss typical daily math instruction, as MMF is meant to supplement the traditional math curriculum and not serve as a replacement. The primary researcher developed a brief fidelity checklist to observe the learning support classroom teacher's typical math instruction and ensured that additional practice with math facts was not provided to students in

the study over what was given with MMF. Fifteen-minute observations were conducted by the primary researcher three times during the course of the study.

Reliability. A second independent observer was trained to independently score 30% of each participant's 1-minute single- and mixed-skills CBMs retrieved from www.aimsweb.com and 2-minute MMF timings to ensure the primary researcher scored all measures correctly as dc/m or dc/2m in the case of 2-minute MMF timings. After independently scoring examples, the first author and trained observer compared scores to ensure no differences existed that were greater than plus or minus one correct digit. Exact agreement as defined by Johnston and Pennypacker (2009), was calculated as the number of agreements divided by the total number of intervals (opportunities) multiplied by 100, and provided the percentage of agreement between the first author and trained observer.

Efficiency of MMF and Social Validity

Efficiency of MMF. Once students moved from baseline data collection phase into intervention phase, the primary researcher used a stopwatch to track the amount of time each intervention session took per participant. Data were recorded as minutes and seconds and were charted daily on an Excel spreadsheet. Data were collected to determine if MMF is a feasible intervention that can be easily and quickly implemented by classroom teachers, paraprofessionals, or classroom volunteers.

Social validity. Upon completion of the study, students were given a researcher-developed survey consisting of Likert-scale questions and two open-ended questions. The Likert scale ranged from 1 to 5, with 1 indicating "strongly disagree" and 5 indicating "strongly agree" to determine the social validity of implementing such an intervention with students with SLD in mathematics. Open-ended questions included questions such as: What did you like (dislike)

about this math program?, and, Do you think this math program helped you learn your math facts? Why or why not?

Chapter 3

Results

To determine the effectiveness of MMF as a supplemental intervention for students with SLD, the following four research questions guided this study:

1. Can students with SLD in mathematics reach expected grade-level single-skill fluency goals (e.g., >50 dc/m) using MMF as a supplemental intervention over an 8-week intervention period?
2. Does the mixed-skills performance of students with SLD in mathematics improve as a result of MMF?
3. Is the use of MMF an efficient intervention for teachers to use with students with SLD in mathematics?
4. How socially valid and acceptable is MMF as intervention for students with SLD in mathematics?

A multiple probe design was implemented to investigate the effectiveness of using MMF to compute basic subtraction facts for students with SLD. Data included performance during baseline, intervention, and maintenance phases and were graphed for all four students. One-minute single-skill CBMs were used to determine stability in the baseline phase. Data were analyzed for changes in levels and trend as evident through visual inspection following implementation of the MMF intervention and are now discussed. Figure 2 displays results for daily 1-minute single-skill CBMs for all four students across baseline, intervention, and maintenance phases. Figure 3 displays results for weekly 2-minute MMF timings for all four students across baseline, intervention, and maintenance phases. Figure 4 displays results for weekly 1-minute mixed-skills CBMs for all four students across baseline, intervention, and

maintenance phases. Table 3 shows descriptive statistics for each student's baseline, intervention, and maintenance for dc/m and errors/minute on 1-minute single-skill CBMs. Table 4 shows descriptive statistics for each student's baseline, intervention, and maintenance for dc/2m and errors/2-minutes on 2-minute MMF timings. Table 5 shows descriptive statistics for each student's baseline, intervention, and maintenance for dc/m and errors/minute on 1-minute mixed-skills CBMs. Due to Alexa only participating in intervention for one day, standard deviations are not reported for her during intervention phase. Results are now discussed for individual students for each of the three dependent variables.

Cheryl

One-minute single-skill CBMs. On pre-test measures, Cheryl's number of dc/m on one-minute single-skill CBMs was 14 with 4 errors. During baseline, Cheryl's number of dc/m on one-minute single-skill CBMs ranged from 13 to 23, with the number of errors ranging from 0 to 9. Cheryl demonstrated stability after nine baseline sessions over 10 days of data collection due to one absence and then began intervention. Once intervention began, Cheryl's number of dc/m on the single-skill CBMs ranged from 17 to 39, with only three data points below 20 dc/m. Her errors ranged from 0 to 12. Visual inspection of the graphed data shows a rapid and immediate acceleration in the level of dc/m following implementation of MMF. Over time, the intervention altered the trend of her baseline data to an upward trend. Twenty-three of the 27 data points during intervention phase were greater than Cheryl's highest baseline point of 23 dc/m. However, Cheryl's data were variable during intervention phase, especially her number of errors. Cheryl passed through set L of MMF. Data collected during the maintenance phase show Cheryl was able to maintain the growth and upward trend of dc/m she exhibited during intervention. All three of Cheryl's data points during maintenance were higher than any of her baseline data points

prior to intervention, and she averaged 36 dc/m on 1-minute single-skill CBMs throughout maintenance.

Two-minute single-skill MMF timings. On pre-test measures, Cheryl's number of dc/2m on two-minute single-skill MMF timings was 18 with 53 errors. During baseline, Cheryl's number of dc/2m on weekly 2-minute single-skill MMF timings ranged from 22 to 23, with the number of errors ranging from 3 to 6. Cheryl demonstrated stability after two weeks of baseline data collection (nine baseline sessions over 10 days of data collection due to one absence) and then began intervention. Once intervention began, Cheryl's number of dc/2m on the 2-minute single-skill MMF timings ranged from 20 to 27, with the number of errors ranging from 1 to 16. Four of the six data points during intervention phase were greater than or equal to Cheryl's highest baseline point of 23 dc/2m. While visual inspection of graphed data did not immediately show an increase in the level of Cheryl's dc/2m, over time the intervention did alter the trend of her baseline data to a very slight upward trend. Data collected during the maintenance phase show Cheryl was able to maintain the growth and overall upward trend of dc/2m she exhibited during intervention. Two of Cheryl's three data points during maintenance were higher than any of her baseline data points prior to intervention, and she averaged 27.3 dc/2m on 2-minute single-skill MMF timings throughout maintenance.

One-minute mixed-skills CBMs. On pre-test measures, Cheryl's number of dc/m on one-minute mixed-skills CBMs was 9 with 4 errors. During baseline, Cheryl's number of dc/m on weekly mixed-skills CBMs ranged from 8 to 17, with the number of errors ranging from 0 to 3. Cheryl began intervention at the completion of the second week of the study. Once intervention began, Cheryl's number of dc/m on the weekly 1-minute mixed-skills CBMs ranged from 10 to 21, with the number of errors ranging from 0 to 3. Two of the seven data points

during intervention were greater than Cheryl's highest baseline point of 17 dc/m. While visual inspection of graphed data did not immediately show an increase in the level of Cheryl's dc/m on mixed-skills, over time the intervention did alter the trend of her baseline data to a noticeable upward trend. Data collected during the maintenance phase show Cheryl was able to maintain the growth and upward trend of dc/m she exhibited during intervention. All three of Cheryl's data points during maintenance were higher than any of her baseline data points prior to intervention, and she averaged 23 dc/m on 1-minute mixed-skills CBMs throughout maintenance.

Amanda

One-minute single-skill CBMs. On pre-test measures, Amanda's number of dc/m on one-minute single-skill CBMs was 8 with 3 errors. During baseline, Amanda's number of dc/m on one-minute single-skill CBMs ranged from 9 to 20, with the number of errors ranging from 0 to 3. Once Cheryl passed through set C of MMF, Amanda demonstrated stability after 13 baseline sessions over 15 days and then began intervention. Once intervention began Amanda's number of dc/m on the single-skill CBMs ranged from 18 to 34. Her errors ranged from 0 to 2. Visual inspection of the graphed data shows a rapid and immediate acceleration in the level of dc/m following implementation of MMF. Over time, the intervention altered the trend of her baseline data to an upward trend. Twenty-four of the 25 data points during intervention phase were greater than Amanda's highest baseline point of 20 dc/m. Amanda's number of errors was typically zero and she only exhibited errors during two days of data collection while in intervention phase. Amanda passed through set K of MMF. Data collected during the maintenance phase show Amanda was able to maintain the growth and upward trend of dc/m she exhibited during intervention. All three of Amanda's data points during maintenance were

higher than any of her baseline data points prior to intervention, and she averaged 32 dc/m on 1-minute single-skill CBMs throughout maintenance.

Two-minute single-skill MMF timings. On pre-test measures, Amanda's number of dc/2m on two-minute single-skill MMF timings was 24 with 0 errors. During baseline, Amanda's number of dc/2m on weekly 2-minute single-skill MMF timings ranged from 26 to 34, with zero errors consistently demonstrated. Amanda exhibited stability after three weeks of baseline data collection (13 baseline sessions over 15 successive days of data collection) and then began intervention. Once intervention began, Amanda's number of dc/2m on the 2-minute single-skill MMF timings ranged from 25 to 39, with the number of errors ranging from 0 to 2. Four of the seven data points during intervention phase were greater than or equal to Amanda's highest baseline point of 34 dc/2m. While visual inspection of graphed data immediately showed an increase in the level of Amanda's dc/2m followed by a decrease in the level of Amanda's dc/2m, over time the intervention altered the trend of her baseline data to a slight upward trend. Data collected during the maintenance phase show Amanda was able to maintain the growth and upward trend of dc/2m she exhibited during intervention. All three of Amanda's data points during maintenance were higher than any of her baseline data points prior to intervention, and she averaged 45 dc/2m on 2-minute single-skill MMF timings throughout maintenance.

One-minute mixed-skills CBMs. On pre-test measures, Amanda's number of dc/m on one-minute mixed-skills CBMs was 12 with 0 errors. During baseline, Amanda's number of dc/m on weekly mixed-skills CBMs ranged from 11 to 13, with the number of errors ranging from 0 to 1. Amanda began intervention at the completion of the third week of the study. Once intervention began, Amanda's number of dc/m on the weekly 1-minute mixed-skills CBMs

ranged from 16 to 25, with the number of errors consistently at zero. All six data points during intervention were greater than Amanda's highest baseline point of 13 dc/m. Visual inspection of the graphed data shows a rapid and immediate acceleration in the level of dc/m following implementation of MMF. Over time, the intervention altered the trend of her baseline data to a noticeable upward trend. Data collected during the maintenance phase show Amanda was able to maintain the growth and upward trend of dc/m she exhibited during intervention. All three of Amanda's data points during maintenance were higher than any of her baseline data points prior to intervention, and she averaged 24.7 dc/m on 1-minute mixed-skills CBMs throughout maintenance.

Marilyn

One-minute single-skill CBMs. On pre-test measures, Marilyn's number of dc/m on one-minute single-skill CBMs was 8 with 3 errors. During baseline, Marilyn's number of dc/m on one-minute single-skill CBMs ranged from 6 to 14, with the number of errors ranging from 0 to 1. Once Amanda passed through set C of MMF, Marilyn demonstrated stability after 18 baseline sessions over 23 days and then began intervention. Once intervention began, Marilyn's number of dc/m on the single-skill CBMs ranged from 8 to 24, with the number of errors ranging from 0 to 2. Excluding the first day of MMF implementation, visual inspection of the graphed data shows an overall acceleration in the level of Marilyn's dc/m following implementation of MMF. Over time, the intervention altered the trend of her baseline data from a flat trend to an upward trend. Fourteen of the 17 data points during intervention phase were greater than Marilyn's highest baseline point of 14 dc/m. Marilyn passed through set C of MMF. Data collected during the maintenance phase show Marilyn was able to maintain the growth and upward trend of dc/m she exhibited during intervention. All three of Marilyn's data points

during maintenance were higher than any of her baseline data points prior to intervention, and she averaged 18 dc/m on 1-minute single-skill CBMs throughout maintenance.

Two-minute single-skill MMF timings. On pre-test measures, Marilyn's number of dc/2m on two-minute single-skill MMF timings was 9 with 6 errors. During baseline, Marilyn's number of dc/2m on the weekly 2-minute single-skill MMF timings ranged from 10 to 21, with the number of errors ranging from 0 to 2. Marilyn demonstrated stability after five weeks of baseline data collection (18 baseline sessions over 23 successive days of data collection) and then began intervention. Once intervention began, Marilyn's number of dc/2m on the 2-minute single-skill MMF timings ranged from 15 to 20, with the number of errors ranging from 0 to 2. None of the data points during intervention phase were greater than or equal to Marilyn's highest baseline point of 21 dc/2m. While visual inspection of graphed data did not immediately show an increase in the level of Marilyn's dc/2m, over time the intervention did alter the trend of her baseline data to a very slight upward trend. Data collected during the maintenance phase show Marilyn was not quite able to maintain the growth of dc/2m she exhibited during intervention. The trend of her maintenance data moved from a very slight upward trend to a relatively flat trend. All three of Marilyn's data points during maintenance were lower than three of her baseline data points prior to intervention, and she averaged 17.3 dc/2m on 2-minute single-skill MMF timings throughout maintenance.

One-minute mixed-skills CBMs. On pre-test measures, Marilyn's number of dc/m on one-minute mixed-skills CBMs was 5 with 3 errors. During baseline, Marilyn's number of dc/m on the weekly mixed-skills CBMs ranged from 1 to 17, with the number of errors ranging from 0 to 2. Marilyn began intervention at the completion of the fifth week of the study. Once intervention began, Marilyn's number of dc/m on the weekly 1-minute mixed-skills CBMs

ranged from 6 to 15, with the number of errors ranging from 0 to 3. None of the data points during intervention were greater than Marilyn's highest baseline point of 17 dc/m. Visual inspection of graphed data did not immediately show an increase in the level of Marilyn's dc/m as there was actually a decrease in the level of dc/m for the first 2 data points following implementation of MMF. However, after Marilyn's level decreased to 6 dc/m, her next 2 data points showed an increase in both level and trend to an upward trend. Data collected during the maintenance phase show Marilyn was able to maintain the growth and upward trend of dc/m she exhibited during intervention. All three of Marilyn's data points during maintenance were higher than all but 1 of her baseline data points prior to intervention, and she averaged 14.7 dc/m on 1-minute mixed-skills CBMs throughout maintenance.

Alexa

One-minute single-skill CBMs. On pre-test measures, Alexa's number of dc/m on one-minute single-skill CBMs was 4 with 1 error. During baseline, Alexa's number of dc/m on the one-minute single-skill CBMs ranged from 4 to 14, with the number of errors ranging from 0 to 3. Once Marilyn passed through set C of MMF, Alexa demonstrated stability after 20 baseline sessions over 39 days and then began intervention. It should be noted that Alexa was absent for a total of 11 days during baseline data collection. Although Alexa was only in intervention phase for one day, once intervention began her number of dc/m on the single-skill CBMs improved to 17 with 0 errors. Visual inspection of the graphed data shows a rapid and immediate acceleration in the level of dc/m following implementation of MMF. The effects of MMF were not observed over time on Alexa's trend as she was only in intervention for one day. The only data point during intervention phase was greater than Alexa's highest baseline point of 14 dc/m. As Alexa was only in intervention for one day, she did not pass any of the MMF

lettered sets. Data collected during the maintenance phase show Alexa was able to maintain the growth of dc/m she exhibited during intervention and that the intervention altered the trend of her baseline data to an upward trend over time. All three of Alexa's data points during maintenance were higher than any of her baseline data points prior to intervention, and she averaged 17.3 dc/m on 1-minute single-skill CBMs throughout maintenance.

Two-minute single-skill MMF timings. On pre-test measures, Alexa's number of dc/2m on two-minute single-skill MMF timings was 4 with 0 errors. During baseline, Alexa's number of dc/2m on the weekly 2-minute single-skill MMF timings ranged from 2 to 8, with the number of errors ranging from 0 to 2. Alexa demonstrated stability after nine weeks of data collection (20 baseline sessions over 39 days of data collection) and then began intervention for one day. Once intervention began, Alexa's number of dc/2m on the 2-minute single-skill MMF timing was 13 with 1 error. The only data point during intervention phase was greater than Alexa's highest baseline point of 8 dc/2m. Visual inspection of graphed data shows a rapid and immediate acceleration in the level of dc/2m following implementation of MMF. The effects of MMF were not observed over time on Alexa's trend as she was only in intervention for one day. Data collected during the maintenance phase show Alexa was able to maintain the growth of dc/2m she exhibited during intervention and that the intervention altered the trend of her baseline data to an upward trend over time. All three of Alexa's data points during maintenance were higher than any of her baseline data points prior to intervention, and she averaged 14 dc/2m on 2-minute single-skill MMF timings throughout maintenance.

One-minute mixed-skills CBMs. On pre-test measures, Alexa's number of dc/m on one-minute mixed-skills CBMs was 5 with 2 errors. During baseline, Alexa's number of dc/m on the weekly mixed-skills CBMs ranged from 3 to 7, with the number of errors ranging from 0

to 2. Alexa began intervention at the completion of the ninth week of the study. Once intervention began, Alexa's number of dc/m on the weekly 1-minute mixed-skills CBMs was 10 with 0 errors. The only data point during intervention was greater than Alexa's highest baseline score of 7 dc/m. Visual inspection of graphed data shows a rapid and immediate acceleration in the level of dc/m on Alexa's mixed-skills performance following implementation of MMF. Again, the effects of MMF were not observed over time on Alexa's trend as she was only in intervention for one day. Data collected during the maintenance phase show Alexa was able to maintain the growth of dc/m she exhibited during intervention and that the intervention altered the trend of her baseline data to an upward trend over time. All three of Alexa's data points during maintenance were higher than any of her baseline data points prior to intervention, and she averaged 14.7 dc/m on 1-minute single-skill CBMs throughout maintenance.

Procedural Fidelity and Reliability

Procedural fidelity. Procedural fidelity was assessed for implementation of the MMF program as implemented by the primary researcher. An independent observer was trained by the primary researcher in required steps for successful implementation of MMF and was instructed on how to complete a researcher-created procedural fidelity checklist. For both baseline data collection and intervention phases, procedural fidelity was 100% across all students, and the second observer noted the primary researcher followed protocol in all instances.

The primary researcher also ensured students only received instruction in math facts during intervention time and that business-as-usual instruction was what occurred during typical math instruction in the students' classroom. Fifteen-minute observations were conducted by the primary researcher three times during the course of the study. Results from the observations

showed students did not receive any additional practice with math facts other than what was provided throughout the study.

Reliability. Reliability was assessed for number of dc/m and dc/2m for all students across both baseline data collection and intervention phases, and was assessed using permanent product agreement. A second observer was trained to score all dependent measures as dc/m or dc/2m and was enlisted to score 30% of all measures. Exact agreement as defined by Johnston and Pennypacker (2009) was calculated as the number of agreements divided by the total number of intervals (opportunities) multiplied by 100 to provide the percentage of agreement between the first author and trained observer. Percentage of agreement on the number of dc/m and dc/2m was 98% across all four students for baseline data collection and intervention phases.

Efficiency of MMF and Social Validity

Efficiency of MMF. Once students moved from baseline data collection phase into intervention phase, the primary researcher used a stopwatch to track the amount of time each intervention session took per participant. Data were collected to determine if MMF is a feasible intervention that can be easily and quickly implemented by classroom teachers, paraprofessionals, or classroom volunteers. Total time of intervention ranged from 7 minutes (Alexa) to 210 minutes (Amanda), with an overall average of 119 total minutes of intervention per student. Per session, time of intervention ranged from an average of 5.5 minutes (Cheryl) to an average of 8 minutes (Amanda), with both Marilyn and Alexa averaging 7 minutes per session.

Social validity. Social validity for the MMF intervention was based on how included participants responded to a brief survey presented to them upon completion of the study. Likert-scale questions included questions such as, “Learning math facts is important”, “Practice with

MMF helped me to learn my math facts”, and “I like how math facts were taught to me when I worked with” the primary researcher. Two open-ended questions were, “What did you like about MMF? What did you dislike?” and “Do you think this math program helped you learn your math facts? Why or why not?” Results from the social validity questions showed all four participants strongly agreed with all Likert-scale questions. Responses from open-ended questions included responses such as, “I liked coloring in the rocket”, “I think it was awesome”, “Yes, because it help me do math at my class”, and “I did not like when I did not pass a set.” Overall positive responses were gathered from all participants about their liking of MMF; the only part they did not like was if they did not pass a set and did not get to color in their rockets.

Results across Participants

Although participants in this study did not reach expected grade-level single-skill fluency goals of greater than 50dc/m, improvements were noted across all students’ dc/m and dc/2m scores from their pre-test and baseline measures following implementation of MMF (see Tables 3, 4, and 5). Overall increases in levels of dc/m on single-skill probes were demonstrated for all four students across baseline data collection to intervention phases. Similarly, increases in levels of dc/2m were evident for three of four participants on weekly 2-minute timings. While visual improvements were not evident for Marilyn, her mean dc/2m was slightly higher following implementation of MMF as compared to her baseline mean dc/2m. Performance on mixed-skills probes (dc/m) also increased for three of four participants following implementation of MMF. Again, although visual improvements were not evident for Marilyn, her mean dc/m was slightly higher following implementation of MMF as compared to her baseline mean dc/m and her maintenance data demonstrated an upward trend of her data over time. MMF was a relatively efficient intervention, taking only about 7-8 minutes per session per student. Four students with

SLD in mathematics who participated in this intervention were positive in their liking of the program and felt it helped them to learn basic math facts in ways that would help them in their daily math class.

Chapter 4

Discussion

The purpose of this study was to investigate the impact of MMF as an intervention on math-fact fluency of students with SLD in mathematics. MMF includes a number of components previously identified as effective practices to instruct students in basic math facts, such as multiple opportunities for practice with modeling, the use of flashcards, instruction in only small sets of four or five facts at one time, and immediate performance feedback. Specifically, the study examined the effectiveness of this intervention for increasing students' fluency as measured by number of digits correct per minute or per 2 minutes when solving basic subtraction problems or mixed basic addition and subtraction problems. All four students demonstrated improvements in their level of dc/m on single-skill probes across baseline and intervention phases. Three of four students demonstrated overall improvements in their level of dc/2m on single-skill 2-minute probes across baseline and intervention phases. The same three students also demonstrated overall increases in their level of dc/m on mixed-skills probes across baseline and intervention phases. This research showed the effectiveness of MMF as a supplemental intervention to improve basic math skills for students with SLD in mathematics who may struggle in the area of basic math facts. This study adds to a small, albeit growing, body of knowledge about effective instructional strategies for students with learning disabilities in the area of mathematics.

Effective instruction in all mathematical areas, including basic math facts, is especially critical for students with SLD and those who may struggle in the learning of basic math facts. Past research has found support for explicit instruction including teacher modeling, close supervision, frequent progress monitoring, and immediate feedback as necessary components to

effectively instruct students with SLD in mathematics (Baker et al., 2002). Similarly, Fuchs et al. (2008) identified seven principles of effective instruction for students with SLD in mathematics including: instructional explicitness, thoughtful instructional design, multiple opportunities for practice, cumulative review over time, and continuing progress monitoring. Swanson (1999) noted direct instruction techniques including teacher modeling, use of small interactive groups, significant opportunities for repetition, and practice-review sessions incorporating daily and/or weekly review activities were all beneficial in helping to remediate academic difficulties for students with SLD in mathematics. Repeated practice has also been identified as an instrumental procedure to promote fluent responding to math problems (Poncy et al., 2010). Mastering Math Facts incorporates many of these key instructional strategies.

A significant benefit of MMF is the use of small manageable sets to instruct students in basic math facts. As students with SLD in mathematics tend to have weaker working memory than children who are not identified with SLD (Sante, McLaughlin, & Weber, 2001), incorporating sets of only two basic math facts plus their inverses is an optimal way to improve students' basic math fact fluency. When instructing struggling students in basic math facts, the goal is to move students from inefficient strategies such as "count all" or "count on" to more efficient and fluent strategies (Ashcraft & Christy, 1995). Hence, implementing an intervention such as MMF promotes basic math fact fluency through the use of direct instruction, repeated and frequent opportunities for repetition and practice, cumulative review involving newly and previously learned facts, ongoing progress monitoring, and immediate and corrective feedback (Baker et al., 2002; Fuchs et al., 2008; Swanson, 1999). Furthermore, in MMF, students are only expected to learn four or five new facts at one time before introduction to other facts.

The addition of MMF as a supplemental intervention for four students with SLD in mathematics proved to be an effective strategy for increasing students' fluency levels in terms of dc/m or dc/2m. Although students did not reach expected fluency levels of greater than 50 dc/m, all four students demonstrated significant improvements on daily single-skill probes from pre-test and baseline measures to intervention measures. A functional relationship was identified and replicated three times on the effects of four students' mathematics performance after implementation of MMF. There was an immediate positive change in level of dc/m for three of four students following implementation of MMF. The fourth student (Marilyn) demonstrated a positive change in level of dc/m after completing two sessions of intervention. Once Marilyn became more familiar with the program, her level of dc/m was almost consistently greater than her level of dc/m during baseline data collection phase.

Amanda and Alexa both demonstrated immediate increases in their level of dc/2m when administered 2-minute weekly timings included with the MMF program across baseline and intervention phases. After the first week of intervention when her level of dc/2m decreased slightly, Cheryl then displayed an overall increase in her level of dc/2m across baseline and intervention phases. Marilyn's overall trend of dc/2m remained relatively flat over time, and her level of dc/2m did not increase over her baseline levels. This could be attributed to the fact that Marilyn was very concerned about her handwriting and spent more time worrying about correctly forming the written answers than actually trying to answer the problems. Marilyn mentioned to the primary researcher that because she had 2-minutes on these measures, she thought that she had enough time to erase her answers and improve upon her writing. Marilyn also complained that the empty space beneath each problem on 2-minute timings was not large enough for her to properly write her numerals. The primary researcher tried to address this issue

by allowing Marilyn to respond orally to 1- and 2-minute probes, but Marilyn became overly agitated and distracted when trying to respond in this manner; therefore, responding in writing continued to be the mode of answering for Marilyn.

On 1-minute mixed-skills probes, Amanda and Alexa both demonstrated immediate increases in their level of dc/m when administered 1-minute weekly probes across baseline and intervention phases. During baseline, Amanda and Alexa both answered problems in the order in which they appeared on the 1-minute probes despite instructions to skip problems with which they were not familiar. Once MMF intervention began, both students realized it was quicker to just answer basic subtraction problems that they knew and began skipping the addition problems. Although Cheryl's level of dc/m on mixed-skills probes initially decreased once intervention began, her final level during intervention was significantly higher than her first baseline point. Marilyn's level of dc/m on 1-minute mixed-skills probes similarly decreased for the first 2 data points after intervention began, but her last 2 data points showed increases in her level of dc/m. Much of Marilyn's data can be attributed to her demeanor for the day; some days she was sullen and refused to work at the level expected by the primary researcher, other days she was extremely talkative and very easily distracted which led to decreases in her levels of dc/m or dc/2m.

Over time, all four students demonstrated sustained knowledge of facts learned during MMF intervention as evidenced by maintenance checks two, three, and four weeks following completion of the study. All four students improved or maintained their average dc/m on daily single-skill CBMs from intervention to maintenance. Cheryl and Amanda significantly improved upon their average dc/m from the end of intervention to their average dc/m after maintenance, while Marilyn and Alexa showed a slight improvement from their average dc/m at

the end of intervention to their average dc/m after maintenance. Similarly, Cheryl and Amanda significantly improved their average dc/2m from the end of intervention until after maintenance as evidenced by 2-minute MMF timings. Alexa also displayed an upward trend in her dc/2m from the end of intervention after maintenance. Marilyn's dc/2m decelerated slightly from the end of intervention until the end of maintenance. All four students improved upon their average of dc/m on mixed-skills CBMs from intervention to maintenance and all demonstrated upward trends of data from the end of intervention through maintenance.

All four students also provided positive responses when asked their feelings about using MMF, which indicates the positive social validity of using MMF as a supplemental intervention to instruct students with SLD in basic math facts. The students found MMF useful and helpful to the learning of basic subtraction facts and were overall very favorable about the program. Daily time of implementation was relatively quick, ranging from an average of 5.5 minutes to 8 minutes per session per student. Classroom teachers could use this time-frame as a guideline when determining if MMF could be a feasible intervention for their classrooms.

Limitations and Future Research

One limitation of this study was the fact that the participants had previously been exposed to Touch Math and had a difficult time just knowing the facts. At least two students (Cheryl and Amanda) would often count the "dots" on numerals, as instructed to do in Touch Math, as opposed to just memorizing the facts and writing the answers down. The other two students would occasionally count the dots as well, but not as often as Cheryl and Amanda. The primary researcher attempted to combat this prior knowledge by often saying, "Don't count, just know the answers" during the 1- and 2-minute timings. However, video clips of two students showed they continued to count the dots as intervention progressed, despite the fact that they did not

count the dots when looking at flashcards or participating in the bottom half of MMF sheets.

The primary researcher believes this practice may have limited the number of digits correct per minute that could be achieved by the students in this study.

A second limitation was that although the primary researcher modeled new facts and the learners had ample opportunities to exhibit the behavior of practicing basic facts, the researcher did not include motivation or reinforcement opportunities other than stickers or stamps at the end of each session. For three of the four students, this delayed reinforcement seemed to be enough motivation to encourage them to work. However, as previously mentioned, Marilyn's performance seemed to depend on her mood for the day, and maybe additional reinforcement or motivation opportunities could have led to increased performance from Marilyn. The primary researcher did discuss this with the classroom teacher and an additional reinforcer in terms of an extra snack was offered to Marilyn one day, but it did not make a difference to her performance, so it was not offered again. The primary researcher could have been more proactive from the start of the study with reinforcement or motivation other than just stickers. Perhaps building in a specific criterion for reinforcement (e.g., a certain number of daily dc/m) may have encouraged all four students to improve their mathematics performance even more than they did once MMF intervention began. Future research could investigate the addition of more structured reinforcement or motivation opportunities when coupled with implementation of MMF to determine if effects of MMF are strengthened when partnered more with extrinsic reinforcers.

A third limitation includes generalizability of results across settings and across change agents. This research was conducted with a small group of students in a learning support classroom; it is uncertain whether the same results would occur if MMF was implemented with students with SLD receiving instruction inside a general education setting or if implemented with

a larger number of students, perhaps entire classrooms. As MMF was not designed specifically for students with disabilities and was in fact intended for use with entire general education classrooms, more research should be conducted to determine if MMF truly is a feasible and effective intervention to use with students with SLD or those who struggle in mathematics. In a similar vein, the primary researcher was the one who implemented the MMF intervention, as opposed to the students' classroom teacher or paraprofessionals in the classroom. To help bridge the research to practice gap, future research should involve trained teachers or paraprofessionals as change agents to implement MMF, rather than just outside researchers, to potentially strengthen effects that were demonstrated in this current study. It is only through further replication of studies such as this that will enable the field to identify MMF as an evidence-based practice for use with students with SLD or those at-risk for a disability in the area of mathematics.

Implications for Teachers

The teaching of basic mathematics skills such as the memorization of math facts is a vital component of elementary school years. Between 5% and 8% of school-age children experience some form of SLD in mathematics that compromises their ability to learn and recall basic facts from memory. With 390 basic facts for all students to learn by the end of Grade 4 (CCSSI, 2010), proficiency with basic math facts continues to emerge as a growing concern for school-age students and beyond. Of particular concern are those students with or at-risk for a learning disability in mathematics who require frequent opportunities to practice and receive immediate feedback on such basic math facts as critical components to their success. Finding effective interventions that can be easily implemented by teachers or paraprofessionals is crucial to students becoming fluent with basic math facts.

This study demonstrates that MMF is an effective intervention for solving basic subtraction facts for students with SLD in mathematics. The students in this study were able to improve their math-fact fluency after implementation of MMF across only 10 weeks of instruction. This study adds to the necessary body of research supporting direct instruction of math facts, particularly for students with disabilities. For students who struggle with basic computational skills, teachers may want to consider the use of MMF as an instructional strategy to help their students reach proficiency by the end of Grade 4 with 390 basic facts.

Mastering Math Facts affords students multiple opportunities to respond to and practice both new and previously learned facts in order to help students build fluency with the solving of basic math facts. Students are provided with immediate and corrective feedback when necessary to ensure correct practice of such facts. Students are only expected to learn two new facts, their inverses, and possibly a doubles fact at one time, which is a more manageable set of facts than attempting to learn facts in groups of 10 or more. With the use of MMF, students are not expected to move on to new facts until mastery has occurred, thus preventing confusion that typically occurs when students are asked to move on too quickly. Implementing an intervention such as MMF promotes basic math fact fluency through the use of direct instruction, repeated and frequent opportunities for repetition and practice, cumulative review involving newly and previously learned facts, ongoing progress monitoring, and immediate and corrective feedback.

The current study indicates MMF is not too time consuming of an intervention, can be implemented with relatively inexpensive materials, and involves direct instruction procedures that have been identified as the most effective ways to teach basic math facts to students with a SLD or at-risk for developing a SLD in mathematics. Implementation of MMF was effective in increasing the subtraction-fact fluency of four students with SLD in mathematics. The materials

used were readily available (i.e., flashcards), inexpensive, and implementation did not require an excessive amount of time (i.e., only 7-8 minutes per day) to be taken away from daily classroom instruction. Therefore, MMF is one intervention that should be considered for use in general or special education classrooms to ensure students reach expected proficiency levels with basic math facts.

Conclusions

In summary, this study examined the impact of MMF as an intervention on math-fact fluency of students with SLD in mathematics. All four students were successful using the program to solve basic subtraction problems and mixed-skills basic addition and subtraction facts. Basic math facts instruction for struggling students should include direct instruction procedures, multiple opportunities for students to practice, frequent and cumulative practice, and immediate and specific feedback. Flash-card type interventions such as MMF are one type of focused intervention to help students become more fluent with basic math facts and enable them to move away from inefficient strategies upon which they may rely. In this study, overall increases were evident in subtraction fact-fluency skills after less than 40 days of implementation, yet more research is needed to determine if the effects are strengthened with a longer intervention period.

References

- AimswEB Mathematics Computation: Administration and Technical Manual. (2010).
Downloaded from <http://www.aimsweb.com/uploads/M-COMP%20Manual.pdf>
- Ashcraft, M.H., & Christy, K.S. (1995). The frequency of arithmetic facts in elementary texts: Addition and multiplication in grades 1-6. *Journal for Research in Mathematics Education*, 26(5), 396-421.
- Baker, S., Gersten, R., & Lee, D.-S. (2002). A synthesis of empirical research on teaching mathematics to low-achieving students. *The Elementary School Journal*, 103(1), 51-73.
- Bryant, D.P., Hartman, P., & Kim, S.A. (2003). Using explicit and strategic instruction to teach division skills to students with learning disabilities. *Exceptionality*, 11(3), 151-164.
- Burns, M.K., VanDerHeyden, A.M., & Jiban, C. (2006). Assessing the instructional level for mathematics: A comparison of methods. *School Psychology Review*, 35, 401-418.
- Cihak, D.F., & Grim, J. (2008). Teaching students with autism spectrum disorder and moderate intellectual disabilities to use counting-on strategies to enhance independent purchasing skills. *Research in Autism Spectrum Disorders*, 2, 716-727.
- Common Core State Standards Initiative. (2010). *Common core state standards for mathematics: Grades K – 8*.
- Crawford, D.B. (2003). *How to Successfully Teach Math Facts to All Students: Workshop Manual*. Eau Claire, WI: Otter Creek Institute.
- Deno, S.L., & Mirkin, P.K. (1977). *Data-based program modification: A manual*. Reston, VA: Council for Exceptional Children.

- Doabler, C.T., Cary, M.S., Jungjohann, K., Clarke, B., Fien, H., Baker, S., . . . Chard, D. (2012). Enhancing core mathematics instruction for students at risk for mathematics disabilities. *Teaching Exceptional Children, 44*(4), 48-57.
- Engelmann, S. & Carnine, D. (1982). *Theory of instruction*. New York, NY: Irvington.
- Flores, M.M. (2009). Teaching subtraction with regrouping to students experiencing difficulty in mathematics. *Preventing School Failure, 53*(3), 145-152.
- Fuchs, L.S., Fuchs, D., Powell, S.R., Seethaler, P.M., Cirino, P.T., & Fletcher, J.M. (2008). Intensive interventions for students with mathematics disabilities: Seven principles of effective practice. *Learning Disability Quarterly, 31*(2), 79-92.
- Fuson, K.C. (1982). An analysis of the counting-on solution procedure in addition. In T.P. Carpenter, J.M. Moser, & T.A. Romberg (Eds.), *Addition and subtraction: A cognitive perspective* (pp. 67–81). Hillsdale, NJ: Lawrence Erlbaum Associates.
- Garnett, K. (1992). Developing fluency with basic number facts: Intervention for students with learning disabilities. *Learning Disabilities Research & Practice, 7*, 210-216.
- Geary, D.C. (2004). Mathematics and learning disabilities. *Journal of Learning Disabilities, 37*(1), 4-15.
- Geary, D.C., Bow-Thomas, C. C., Liu, F., & Siegler, R.S. (1996). Development of arithmetical competencies in Chinese and American children: Influence of age, language, and schooling. *Child Development, 67*(5), 2022-2044.
- Geary, D.C., Boykin, A.W., Embretson, S., Reyna, V., Siegler, R.S., Berch, D.B., & Graban, J. (2008). Report of the task group on learning processes. In *Foundations for Success: The Final Report of the National Mathematics Advisory Panel*, U.S. Department of Education: Washington, DC.

- Gersten, R., Beckmann, S., Clarke, B., Foegen, A., Marsh, L., Star, J. R., & Witzel, B. (2009). *Assisting students struggling with mathematics: Response to Intervention (RtI) for elementary and middle schools* (NCEE 2009-4060). Washington, DC: National Center for Education Evaluation and Regional Assistance, Institute of Education Sciences, U.S. Department of Education. Retrieved from:
http://ies.ed.gov/ncee/wwc/pdf/practice_guides/rti_math_pg_042109.pdf
- Gersten, R., & Chard, D. (1999). Number sense: Rethinking arithmetic instruction for students with mathematical disabilities. *Journal of Special Education, 33*(1), 18-28.
- Griffin, S., & Case, R. (1997). Wrapping up: Using peer commentaries to enhance models of mathematics teaching and learning. *Issues in Education, 3*, 115-134.
- Haring, N.G., & Eaton, M.D. (1978). Systematic instructional procedures: An instructional hierarchy. In N. G. Haring, T. C. Lovitt, M. D. Eaton, & C. L. Hansen (Eds.), *The fourth r: Research in the classroom* (pp. 23-40). Columbus, OH: Charles E. Merrill Publishing Co.
- Hasselbring, T.S., Goin, L., & Bransford, J. D. (1988). Developing math automaticity in learning handicapped children: The role of computerized drill and practice. *Focus on Exceptional Children, 20*, 1-7.
- Hosp, M.K., Hosp, J.L., & Howell, K.W. (2007). *The ABCs of CBM: A practical guide to curriculum-based measurement*. New York, NY: The Guilford Press.
- Johnston, J.M., & Pennypacker, H.S. (2009). *Strategies and tactics of behavioral research*. New York, NY: Routledge.

- Jordan, N.C., Kaplan, D., & Hanich, L.B. (2002). Achievement growth in children with learning difficulties in mathematics: Findings of a two-year longitudinal study. *Journal of Educational Psychology, 94*, 586-597.
- Kroesbergen, E.H., & Van Luit, J.E. (2003). Mathematics interventions for children with special educational needs: A meta-analysis. *Remedial and Special Education, 24*(2), 97-114.
- Lin, F., & Kubina, R.M., Jr. (2005). A preliminary investigation of the relationship between fluency and application for multiplication. *Journal of Behavioral Education, 14*, 73-87.
- McClelland, M.M., Acock, A.C., & Morrison, F.J. (2006). The impact of kindergarten learning related skills on academic trajectories at the end of elementary school. *Early Childhood Research Quarterly, 21*, 471-490.
- Miller, A.D., & Heward, W.L. (1992). Do your students really know their math facts? Using daily time trials to build fluency. *Intervention in School and Clinic, 28*(2), 98-104.
- Miller, K.C., Skinner, C.H., Gibby, L., Gaylon, C.E., & Meadows-Allen, S. (2011). Evaluating generalization of addition-fact fluency using the taped-problems procedures in a second-grade classroom. *Journal of Behavioral Education, 20*, 203-220.
- Miller, S., & Mercer, C. (1997). Educational aspects of mathematics disabilities. *Journal of Learning Disabilities, 30*, 47-56.
- Morgan, P.L., Farkas, G., & Wu, Q. (2011). Kindergarten children's growth trajectories in reading and mathematics: Who falls increasingly behind? *Journal of Learning Disabilities, 44*(5), 472-488.
- Murphy, R.J., & Bryan, A.J. (1980). Multiple-baseline and multiple-probe designs: Practical alternatives for special education assessment and evaluation. *Journal of Special Education, 14*(3), 325-335.

- National Council of Teachers of Mathematics. (2006). *Curriculum focal points for prekindergarten through grade 8 mathematics: A quest for coherence*. Reston, VA: National Council of Teachers of Mathematics. Retrieved August 1, 2007 from <http://www.nctm.org/standards/focalpoints.aspx?id=298>.
- National Mathematics Advisory Panel. (2008). *Foundations for Success: The Final Report of the National Mathematics Advisory Panel*, U.S. Department of Education: Washington, DC.
- Pellegrino, J. W., & Goldman, S. R. (1987). Information processing and elementary mathematics. *Journal of Learning Disabilities*, 20, 23–32, 57.
- Poncy, B.C., Duhon, G.J., Lee, S.B., & Key, A. (2010). Evaluation of techniques to promote generalization with basic math fact skills. *Journal of Behavioral Education*, 19, 76-92.
- Poncy, B.C., Skinner, C.H., & Jaspers, K.E. (2007). Evaluating and comparing interventions designed to enhance math fact accuracy and fluency: Cover, copy, and compare versus taped problems. *Journal of Behavioral Education*, 16, 27-37.
- Rivera-Batiz, F.L. (1992). Quantitative literacy and the likelihood of employment among young adults in the United States. *The Journal of Human Resources*, 27, 313-328.
- Sante, A.D., McLaughlin, T.F., & Weber, K.P. (2001). The use and evaluation of a direct instruction flash card strategy on multiplication math facts mastery with two students with developmental disabilities and attention deficit hyperactivity disorder. *Journal of Precision Teaching & Celeration*, 17(2), 68-75.
- Sidman, M. (1960). *Tactics of scientific research: Evaluating experimental data in psychology*. Boston, MA: Authors Cooperative, Inc.
- Smith, K.S., & Geller, C. (2004). Essential principles of effective mathematics instruction: Methods to reach all students. *Preventing School Failure*, 48(4), 22-29.

- Stein, M., Silbert, J., & Carnine, D. (2006). *Designing effective mathematics instruction: A direct instruction approach* (4th ed.). Upper Saddle River, NJ: Pearson Education.
- Stock, P., Desoete, A., & Roeyers, H. (2010). Detecting children with arithmetic disabilities from kindergarten: Evidence from a 3-year longitudinal study on the role of preparatory arithmetic abilities. *Journal of Learning Disabilities, 43*(3), 250-268.
- Swanson, H.L. (1999). Instructional components that predict treatment outcomes for students with learning disabilities: Support for a combined strategy and direct instruction model. *Learning Disabilities Research & Practice, 14*(3), 129-140.
- Swanson, H.L., & Sachse-Lee, C. (2000). A meta-analysis of single-subject-design intervention research for students with LD. *Journal of Learning Disabilities, 33*(2), 114-136.
- Vukovic, R.K., & Siegel, L.S. (2010). Academic and cognitive characteristics of persistent mathematics difficulty from first through fourth grade. *Learning Disabilities Research and Practice, 25*(1), 25-38.
- Witzel, B. S., Riccomini, P. J., & Schneider, E. (2008). Implementing CRA math instruction with secondary students with learning disabilities. *Intervention in School and Clinic, 43* (5), 270-276.
- Woodward, J. (2006). Developing automaticity in multiplication facts: Integrating strategy instruction with timed practice drills. *Learning Disability Quarterly, 29*, 269-289.
- www.oci-sems.com (n.d.). Otter Creek Institute Educator Workshops and Supplemental Products.

Appendix A

Tables

Table 1

Participant characteristics

Variable	Cheryl	Participants Amanda	Marilyn	Alexa
Age (years, months)	9 yrs., 4 months	9 yrs., 0 months	8 yrs., 11 months	7 yrs., 11 months
Grade level	3	3	3	2
Gender	Female	Female	Female	Female
Race	Caucasian	Caucasian	African- American	African- American
Exceptionality	SLD in math	SLD in math	SLD in math	SLD in math
How Fast Can You Write?	40	42	28	26
One-Minute Single-skill (dc/m)	14	8	8	4
Two-Minute Single-skill (dc/2m)	18	24	9	4
One-Minute Mixed-skills (dc/m)	9	12	5	5

Note: SLD = specific learning disability.

dc/m = digits correct per minute, dc/2m = digits correct per 2 minutes.

Table 3

Single-Skill Digits Correct and Errors per Minute: Means and Standard Deviations

Student	Phase	Digits Correct per Minute	Errors per Minute
		M (SD)	M (SD)
Cheryl	Baseline	18.7 (3.9)	3.8 (2.9)
	Intervention	26.4 (4.8)	2.4 (2.7)
	Maintenance	36 (3.6)	3.3 (1.2)
Amanda	Baseline	14.8 (3.5)	0.5 (1.0)
	Intervention	28 (3.7)	0.12 (0.44)
	Maintenance	32 (3.6)	0 (0)
Marilyn	Baseline	11.1 (2.2)	0.24 (0.43)
	Intervention	17.6 (4.2)	0.41 (0.7)
	Maintenance	18.7 (0.57)	0 (0)
Alexa	Baseline	9.3 (2.5)	0.8 (1.0)
	Intervention	17 (n/a)	0 (n/a)
	Maintenance	17.3 (1.5)	0.33 (0.57)

Table 4

Single-Skill Digits Correct and Errors per 2-Minutes: Means and Standard Deviations

Student	Phase	Digits Correct per 2-	Errors per 2-Minutes
		Minutes M (SD)	M (SD)
Cheryl	Baseline	22.5 (0.7)	4.5 (2.1)
	Intervention	23.2 (2.6)	6.0 (6.1)
	Maintenance	27.3 (6.7)	3 (1.7)
Amanda	Baseline	29.3 (4.2)	0 (0.0)
	Intervention	32.6 (5.5)	0.7 (0.76)
	Maintenance	45 (6)	0.33 (0.58)
Marilyn	Baseline	16.6 (4.5)	0.8 (1.1)
	Intervention	18.2 (1.9)	0.6 (0.9)
	Maintenance	17.3 (1.2)	0.67 (0.58)
Alexa	Baseline	6 (2.1)	1.2 (0.75)
	Intervention	13 (n/a)	1 (n/a)
	Maintenance	14 (1.7)	0.33 (0.58)

Table 5

Mixed-skills Digits Correct and Errors per Minute: Means and Standard Deviations

Student	Phase	Digits Correct per	Errors per Minute
		Minute	
		M (SD)	M (SD)
Cheryl	Baseline	12.5 (6.4)	1.5 (2.1)
	Intervention	15.1 (3.8)	1.4 (1.3)
	Maintenance	23 (5)	1.7 (0.6)
Amanda	Baseline	12.3 (1.2)	0.3 (0.6)
	Intervention	21.2 (3.9)	0 (0)
	Maintenance	24.7 (3.5)	0 (0)
Marilyn	Baseline	8.2 (6.6)	0 (0)
	Intervention	10 (4.2)	1 (1.4)
	Maintenance	14.7 (1.5)	0.7 (1.2)
Alexa	Baseline	6.4 (1.4)	1(0.5)
	Intervention	10 (n/a)	0 (n/a)
	Maintenance	14.7 (2.5)	0 (0)

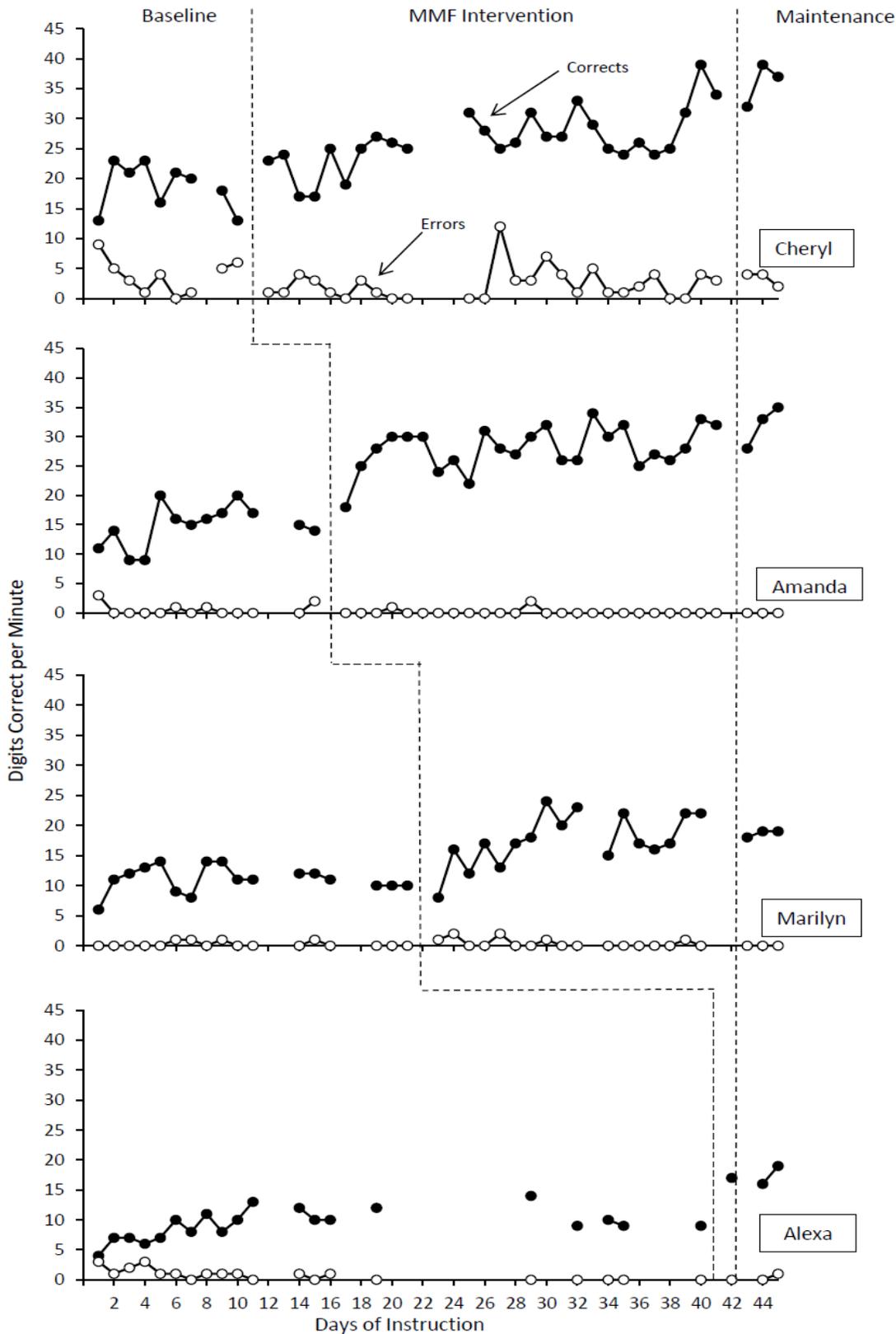


Figure 2. Daily 1-minute single-skill CBMs across baseline, intervention, and maintenance.

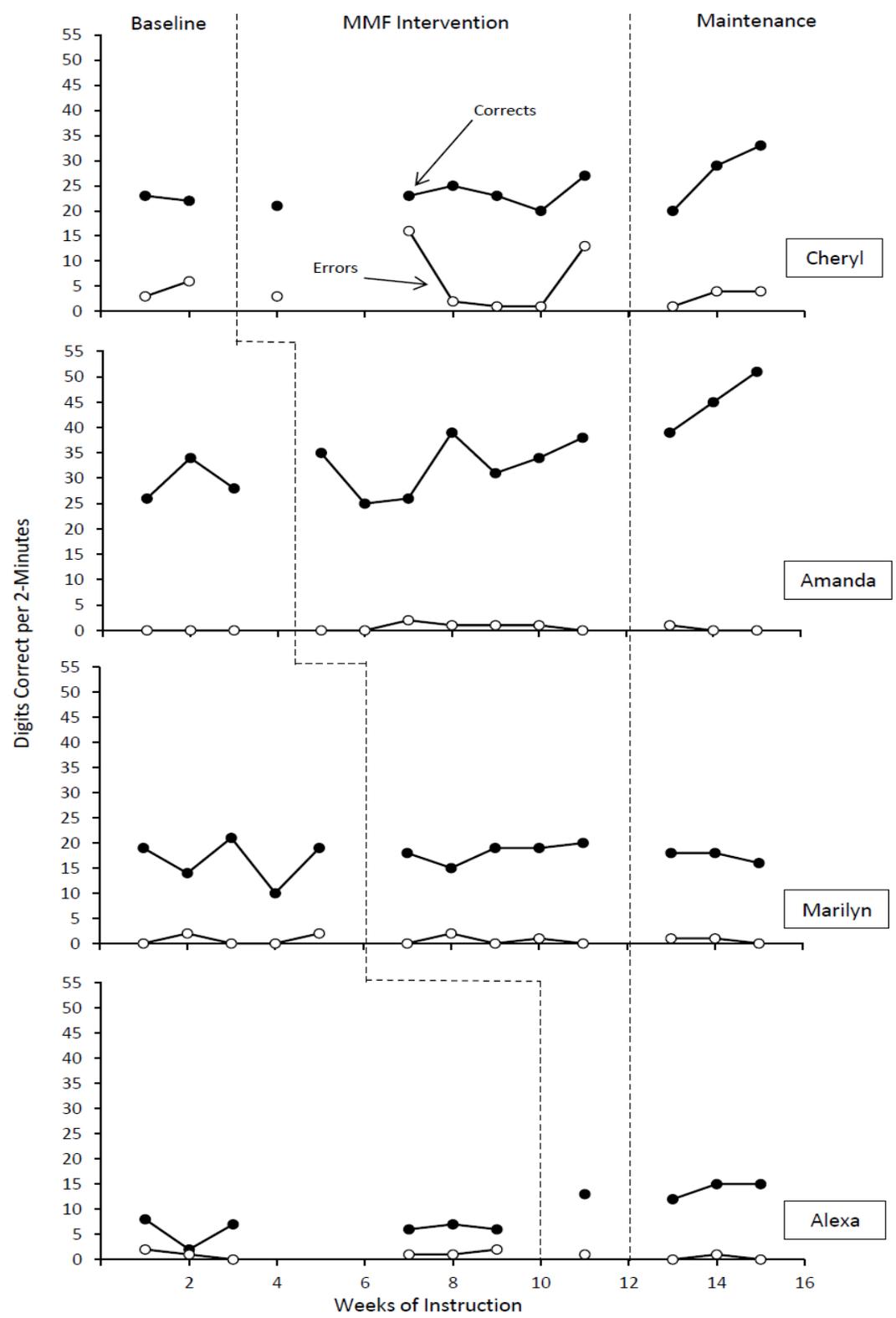


Figure 3. Weekly 2-minute single-skill CBMs across baseline, intervention, and maintenance.

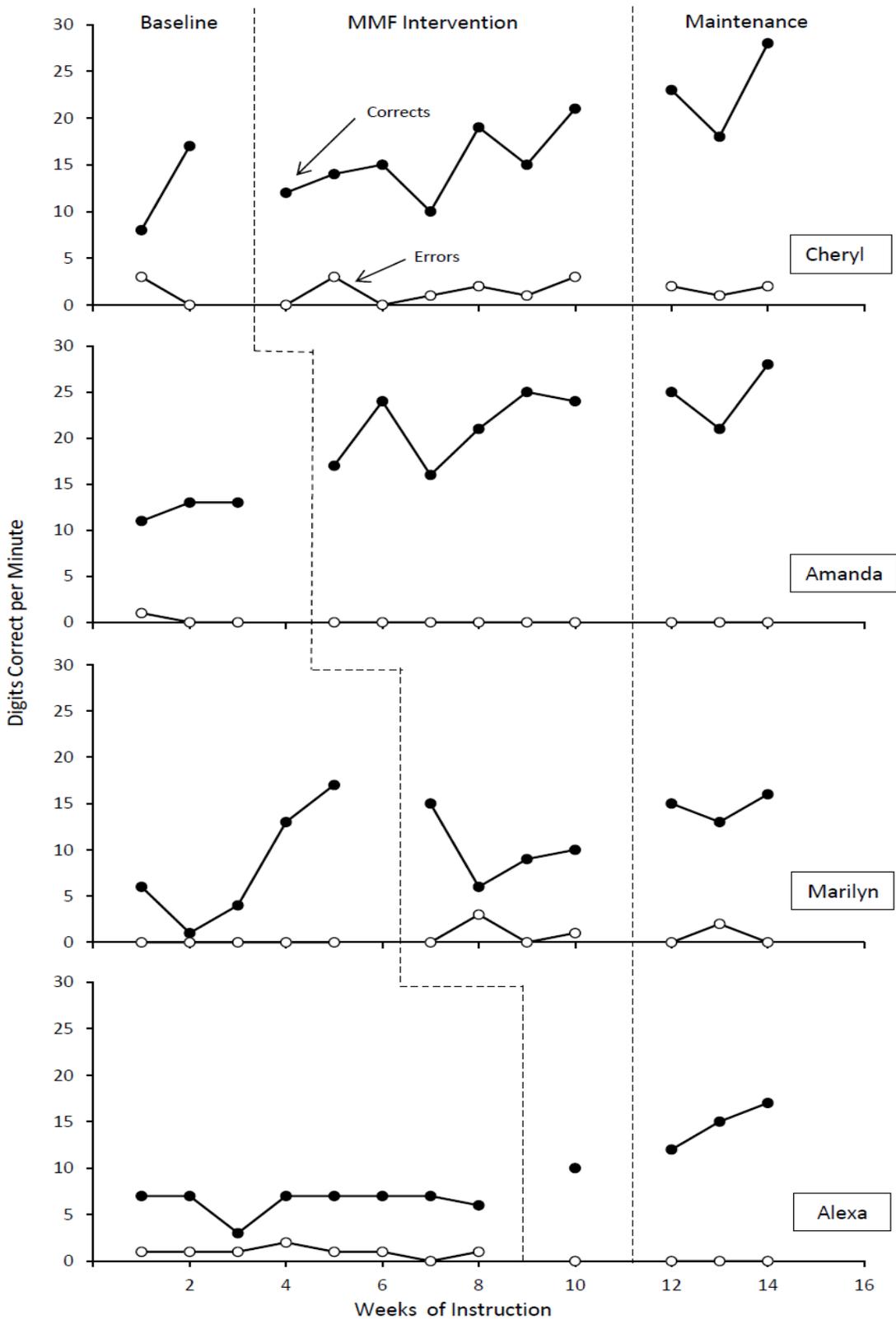


Figure 4. Weekly 1-minute mixed-skills CBMs across baseline, intervention, and maintenance.

Mastering Math Facts - Subtraction S-6 Name _____

Set B [5-4, 5-1, 2-1]

Practice on facts through Set **B**

$$\begin{array}{r} 5 \\ -1 \\ \hline \end{array} \quad \begin{array}{r} 5 \\ -4 \\ \hline \end{array} \quad \begin{array}{r} 3 \\ -2 \\ \hline \end{array} \quad \begin{array}{r} 2 \\ -1 \\ \hline \end{array} \quad \begin{array}{r} 4 \\ -1 \\ \hline \end{array} \quad \begin{array}{r} 5 \\ -4 \\ \hline \end{array} \quad \begin{array}{r} 3 \\ -1 \\ \hline \end{array} \quad \begin{array}{r} 4 \\ -3 \\ \hline \end{array} \quad \begin{array}{r} 5 \\ -1 \\ \hline \end{array} \quad \begin{array}{r} 2 \\ -1 \\ \hline \end{array}$$

$$\begin{array}{r} 3 \\ -1 \\ \hline \end{array} \quad \begin{array}{r} 2 \\ -1 \\ \hline \end{array} \quad \begin{array}{r} 5 \\ -1 \\ \hline \end{array} \quad \begin{array}{r} 4 \\ -1 \\ \hline \end{array} \quad \begin{array}{r} 5 \\ -1 \\ \hline \end{array} \quad \begin{array}{r} 4 \\ -3 \\ \hline \end{array} \quad \begin{array}{r} 5 \\ -4 \\ \hline \end{array} \quad \begin{array}{r} 3 \\ -1 \\ \hline \end{array} \quad \begin{array}{r} 3 \\ -2 \\ \hline \end{array} \quad \begin{array}{r} 4 \\ -3 \\ \hline \end{array}$$

$$\begin{array}{r} 5 \\ -1 \\ \hline \end{array} \quad \begin{array}{r} 5 \\ -4 \\ \hline \end{array} \quad \begin{array}{r} 3 \\ -1 \\ \hline \end{array} \quad \begin{array}{r} 4 \\ -3 \\ \hline \end{array} \quad \begin{array}{r} 3 \\ -1 \\ \hline \end{array} \quad \begin{array}{r} 5 \\ -1 \\ \hline \end{array} \quad \begin{array}{r} 4 \\ -1 \\ \hline \end{array} \quad \begin{array}{r} 5 \\ -4 \\ \hline \end{array} \quad \begin{array}{r} 3 \\ -1 \\ \hline \end{array} \quad \begin{array}{r} 2 \\ -1 \\ \hline \end{array}$$

$$\begin{array}{r} 2 \\ -1 \\ \hline \end{array} \quad \begin{array}{r} 3 \\ -2 \\ \hline \end{array} \quad \begin{array}{r} 5 \\ -1 \\ \hline \end{array} \quad \begin{array}{r} 4 \\ -3 \\ \hline \end{array} \quad \begin{array}{r} 5 \\ -4 \\ \hline \end{array} \quad \begin{array}{r} 3 \\ -1 \\ \hline \end{array} \quad \begin{array}{r} 5 \\ -1 \\ \hline \end{array} \quad \begin{array}{r} 5 \\ -4 \\ \hline \end{array} \quad \begin{array}{r} 2 \\ -1 \\ \hline \end{array} \quad \begin{array}{r} 3 \\ -2 \\ \hline \end{array}$$

One Minute Timing on facts through Set B

$$\begin{array}{r} 5 \\ -1 \\ \hline \end{array} \quad \begin{array}{r} 4 \\ -1 \\ \hline \end{array} \quad \begin{array}{r} 4 \\ -3 \\ \hline \end{array} \quad \begin{array}{r} 5 \\ -1 \\ \hline \end{array} \quad \begin{array}{r} 5 \\ -4 \\ \hline \end{array} \quad \begin{array}{r} 4 \\ -3 \\ \hline \end{array} \quad \begin{array}{r} 3 \\ -1 \\ \hline \end{array} \quad \begin{array}{r} 4 \\ -1 \\ \hline \end{array} \quad \begin{array}{r} 3 \\ -2 \\ \hline \end{array} \quad \begin{array}{r} 2 \\ -1 \\ \hline \end{array}$$

$$\begin{array}{r} 4 \\ -1 \\ \hline \end{array} \quad \begin{array}{r} 3 \\ -2 \\ \hline \end{array} \quad \begin{array}{r} 3 \\ -1 \\ \hline \end{array} \quad \begin{array}{r} 5 \\ -4 \\ \hline \end{array} \quad \begin{array}{r} 4 \\ -3 \\ \hline \end{array} \quad \begin{array}{r} 2 \\ -1 \\ \hline \end{array} \quad \begin{array}{r} 5 \\ -1 \\ \hline \end{array} \quad \begin{array}{r} 3 \\ -2 \\ \hline \end{array} \quad \begin{array}{r} 3 \\ -1 \\ \hline \end{array} \quad \begin{array}{r} 2 \\ -1 \\ \hline \end{array}$$

$$\begin{array}{r} 3 \\ -1 \\ \hline \end{array} \quad \begin{array}{r} 5 \\ -4 \\ \hline \end{array} \quad \begin{array}{r} 4 \\ -3 \\ \hline \end{array} \quad \begin{array}{r} 5 \\ -1 \\ \hline \end{array} \quad \begin{array}{r} 4 \\ -1 \\ \hline \end{array} \quad \begin{array}{r} 5 \\ -4 \\ \hline \end{array} \quad \begin{array}{r} 2 \\ -1 \\ \hline \end{array} \quad \begin{array}{r} 5 \\ -4 \\ \hline \end{array} \quad \begin{array}{r} 5 \\ -1 \\ \hline \end{array} \quad \begin{array}{r} 4 \\ -1 \\ \hline \end{array}$$

$$\begin{array}{r} 2 \\ -1 \\ \hline \end{array} \quad \begin{array}{r} 5 \\ -4 \\ \hline \end{array} \quad \begin{array}{r} 5 \\ -1 \\ \hline \end{array} \quad \begin{array}{r} 3 \\ -1 \\ \hline \end{array} \quad \begin{array}{r} 3 \\ -2 \\ \hline \end{array} \quad \begin{array}{r} 4 \\ -3 \\ \hline \end{array} \quad \begin{array}{r} 3 \\ -2 \\ \hline \end{array} \quad \begin{array}{r} 2 \\ -1 \\ \hline \end{array} \quad \begin{array}{r} 5 \\ -4 \\ \hline \end{array} \quad \begin{array}{r} 2 \\ -1 \\ \hline \end{array}$$

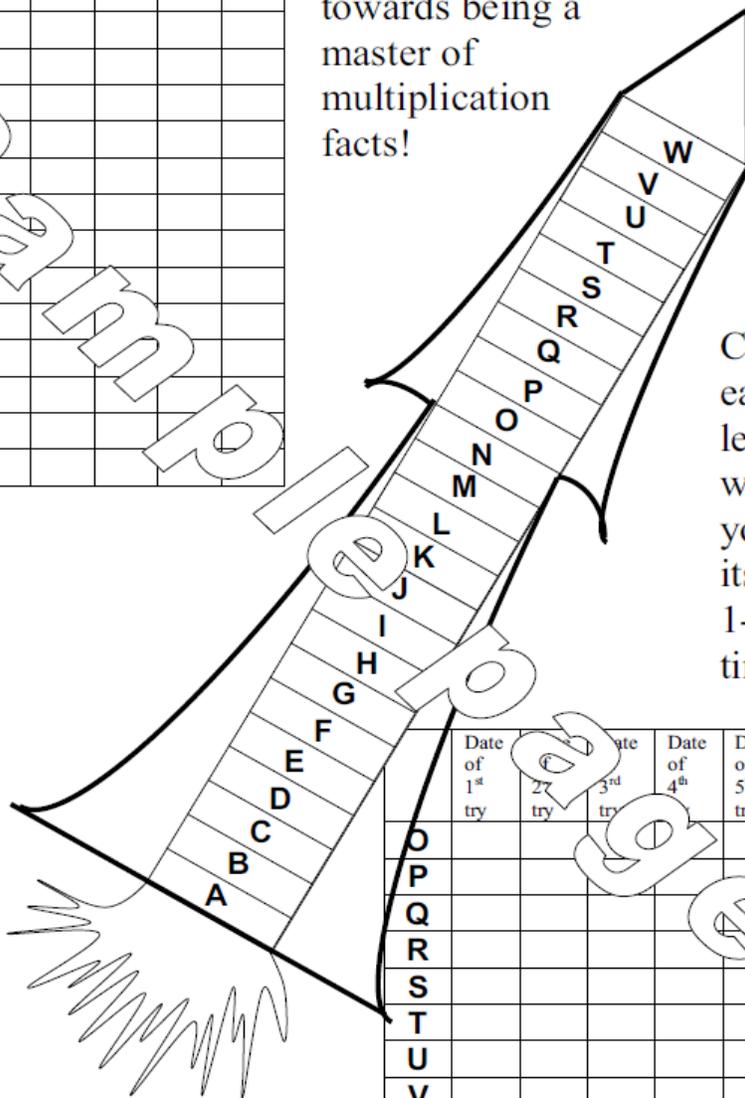
1 minute timing goal _____ Number of problems correct _____

Figure 5. Sample Mastering Math Facts lettered set.

ROCKET CHART

	Date of 1 st try	Date of 2 nd try	Date of 3 rd try	Date of 4 th try	Date of 5 th try	Date of 6 th try
A						
B						
C						
D						
E						
F						
G						
H						
I						
J						
K						
L						
M						
N						

_____ is
ROCKETING
 towards being a
 master of
 multiplication
 facts!



Color in
 each
 letter
 when
 you pass
 its
 1-minute
 timing.

	Date of 1 st try	Date of 2 nd try	Date of 3 rd try	Date of 4 th try	Date of 5 th try	Date of 6 th try
O						
P						
Q						
R						
S						
T						
U						
V						
W						

Figure 6. Sample Mastering Math Facts Rocket Chart.

Appendix C

Review of Relevant Literature

Effects of Basic Math Facts Interventions for Students with Disabilities
or Demonstrating Skills Deficits in Mathematics: A Literature Review

Years of research have demonstrated that students with specific learning disabilities (SLD) in mathematics often experience long-lasting problems when developing fluency with basic math facts, have difficulties when attempting to retrieve those facts from long-term memory, and demonstrate poorer computational skills than students without SLD (Geary, 2004; Mercer & Miller, 1992; Zentall, 1990). Students who do not develop this fluency with basic math facts often lag behind their peers in academic achievement throughout their school years as the performance gap continues to widen (McClelland, Acock, & Morrison, 2006; National Mathematics Advisory Panel, NMAP, 2008). Both the NMAP and the Common Core State Standards Initiative for Mathematics (CCSSI, 2010) expect proficiency from all students with multi-digit multiplication problems as early as the end of Grade 4, with proficiency for single-digit multiplication skills expected as early as the end of Grade 3 (CCSSI, 2010). Figure 1 illustrates specific fluency standards for both Grades 2 and 3.

Figure 1
Fluency Standards for Grades 2 and 3

<p>Grade 2, Operations and Algebraic Thinking Standard 2.OA.2: Fluently add and subtract within 20 using mental strategies. By the end of Grade 2, know from memory all sums of two one-digit numbers (CCSSI, 2010, p. 19).</p>	<p>Grade 3, Operations and Algebraic Thinking Standard 3.OA.7: Fluently multiply and divide within 100. By the end of Grade 3, know from memory all products of two one-digit numbers (CCSSI, 2010, p. 23).</p>
--	--

Stein, Silbert, and Carnine (2006) identify 390 basic math facts all students are expected to learn, which poses a significant challenge to educators to ensure all students are proficient by the end of Grade 3.

Mercer and Miller (1992) note that for students with SLD, math difficulties start early and continue throughout their school careers. Cawley and Miller (1989) found that mathematical learning for students with SLD typically progresses about 1 year for every 2 years the student attends school, which puts students with SLD at a significant disadvantage when attempting to master 390 basic facts in a limited amount of time. Yet, mathematics difficulties are not just a school-related concern. Many activities of daily living call for the application of basic math skills, such as computing scores in games, banking transactions, balancing a checkbook, following schedules, estimating, computing sales tax, solving problems, and understanding measurements in recipes (Mercer & Miller, 1989). Coddling, Hilt-Panahon, Panahon, & Benson (2009) found a strong relationship between computation and application of mathematic skills to overall mathematic success as students progress through school. The mastering of basic math facts, or number combinations, can lead to more success with abstract thinking and problem-solving skills in mathematics, various applications involving money and time management skills, and more complex mathematical tasks including fractions, algebra, and geometry (Coddling, Eckert, Fanning, Shiyko, & Solomon, 2007; Coddling et al., 2009).

Basic Math Facts

A recent report by the NMAP (2008) found that students in the United States cannot solve single-digit addition, subtraction, multiplication, and division facts as rapidly or efficiently as students from other nations. The NMAP believes these differences are due to the amount and quality of practice that happens in American classrooms with regard to computational fluency (Coddling, Burns, & Lukito, 2011). Providing students with adequate number of opportunities to practice basic math facts is essential, both in daily practice and cumulative reviews and practice over time (Daly, Martens, Barnett, Witt, & Olson, 2007). Students with SLD in mathematics

tend to demonstrate low levels of computational fluency that most likely will persist into adulthood, especially if not provided with sufficient practice opportunities to become fluent in basic math facts (Jordan, Hanich, & Kaplan, 2003). Specifically, a recent IES guide recommends instruction for students who struggle in or may need assistance with mathematics should include about 10 minutes of every intervention session to build fluent retrieval of basic facts, and that educators should “consider using technology, flashcards, and other materials for extensive practice to facilitate automatic retrieval,” (Gersten et al., 2009, p. 38).

Components of Effective Instruction in Mathematics

A number of different research teams have conducted reviews to determine components of effective instruction in mathematics for students with SLD in mathematics (e.g., Baker, Gersten, & Lee, 2002; Burns, Coddling, Boice, & Lukito, 2010; Coddling et al., 2009; Coddling et al. 2011; Mercer & Miller, 1992). Many consistent elements have been found across extant reviews. For example, Coddling et al. (2011) recommend providing direct practice on retrieval of basic facts, timed practice with immediate and corrective feedback, and multiple opportunities to recall basic math facts. Mercer and Miller (1992) suggest providing systematic and explicit instruction, monitoring progress, providing feedback, and teaching to mastery. Mercer and Miller, Rosenshine and Stevens (1986), and many other researchers note that explicit instruction should include demonstration or modeling, guided practice with immediate feedback and prompts, and independent practice coupled with feedback.

Similarly, Burns et al. (2010) found support for providing modeling, guided practice, and frequent feedback especially when students are in the acquisition, or first phase, of learning. Burns et al. further specified that students who are in the fluency, or second phase, of learning, should be provided with additional opportunities to practice distinct skills in order to improve

proficiency with certain skills. In a review of simple and moderate intensity interventions, Coddling et al. (2009) identified a number of critical components to address mathematics computation problems, including: drill, repetition, segmentation of skills, active instruction, contingent reinforcement, performance feedback, goal setting, changing the form of practice opportunities, directing the focus of instruction to specific skill weaknesses, increasing the number of opportunities to practice, and/or increasing the pace of instruction.

Instruction in basic facts. Although Kroesbergen and Van Luit's (2003) meta-analysis of mathematics interventions for children with special needs identified aforementioned direct instruction methods as the most effective instructional strategies for students to learn basic math facts, recent reviews in the area of mathematics point out that traditional math programs often fail to provide direct and explicit demonstrations of key skills, frequent opportunities for students to practice target skills, and procedures for providing corrective feedback to students (Doabler et al., 2012). Similarly, students are often expected to learn basic facts in groups of 10 or more, are expected to move on to new facts before mastery has occurred, and are often only provided "instruction" through mad minute assessments, which do not actually teach anything (Crawford, 2003).

Students without disabilities typically progress from a "counting all" strategy (e.g., count all addends separately then counting them together) to a "counting on" strategy (e.g., count on after saying the first addend) to a "counting-on-to-the-larger-addend" strategy (e.g., selecting the larger addend and counting on from there) where "1" is not always the first addend (Ashcraft & Christy, 1995). Students without disabilities are often able to move beyond the use of such strategies and can learn to solve basic math facts without really thinking about the answers (Pellegrino & Goldman, 1987). While these strategies are beneficial for students first learning

basic math facts, continued use of the strategies is inefficient and will limit the amount of energy students have to understand more complex skills such as multi-digit computation problems, fractions, or algebra (Gersten & Chard, 1999).

Flashcard instruction. Past reviews in effective instruction have all emphasized the need for multiple and frequent opportunities for students to practice specific skills (e.g., Burns et al., 2010; Coddling et al., 2009; Coddling et al., 2011; Mercer & Miller, 1992). Repeated practice has been associated with developing automaticity, fluency, and rapid recognition with discrete skills (Volpe, Mule, Briesch, Joseph, & Burns, 2011). Implementation of flashcards is one strategy that affords students more frequent opportunities to practice activities such as sight words (Heron, Heward, Cooke, & Hill, 1983), word recognition (Volpe et al., 2011), and letter sounds (Erbey, McLaughlin, Derby, & Everson, 2011). Similarly, Coddling et al. (2009) found support for the use of flashcards as a simple intensity intervention that could be implemented with skills in mathematics as a way to provide more practice opportunities, specifically for students with SLD in mathematics. Flashcards are easy to implement, do not require a great deal of resources, and can be implemented quickly and easily by teachers, paraprofessionals, parents, or even peers (Nist & Joseph, 2008).

Purpose

Given what is known about effective instruction in mathematics, the purpose of this literature review is to examine current practices for using flashcards as a way of providing instruction to teach basic math facts to students with disabilities or demonstrating skills deficits in mathematics. The following questions guided this review:

1. What components of flashcard instruction increase mathematics performance (e.g., digits correct per minute, number of problems correct, etc.) of students with disabilities or demonstrating skills deficits in mathematics?
2. In what types of settings (e.g., general education, special education) have flashcard interventions been implemented with students with disabilities or demonstrating skills deficits in mathematics? For how much time has flashcard instruction been implemented in order to promote academic achievement in mathematics for students with disabilities or demonstrating skills deficits in mathematics?
3. For what types of disabilities have flashcard interventions demonstrated an increase in mathematics performance of students with disabilities or demonstrating skills deficits in mathematics?

Method

For this literature review, intervention studies had to meet preset inclusionary selection criteria. The criteria for inclusion in this review were: (a) participants were school-age (K-12th grade) and from the United States, (b) the study implemented a basic math facts intervention that utilized flashcards in a direct instruction manner, (c) participants were identified with a disability or as having skills deficits in mathematics, (d) studies were published in peer-reviewed journals, and (e) studies investigated the effect of at least one dependent variable that measured academic outcomes, such as digits correct per minute, percentage of correct responses, or accuracy of problems completed.

A systematic literature search was conducted to identify potential studies for inclusion in this literature review. First, a computer search of three databases, PSYCIInfo, ProQuest Education Journals, and ERIC, was conducted using terms such as: *math instructional*

interventions, basic math facts interventions, math interventions, students with disabilities, students with math disabilities. Searches were limited to studies published between 1989 and November 2012 in order to capture studies reflecting changes made to the standards of the National Council of Teachers of Mathematics (NCTM, 1989). This preliminary electronic search resulted in a total of 108 abstracts of which 87 studies, whose abstracts met the inclusion criteria, were selected for further review. The participant description, dependent variables, and results section were more thoroughly examined to determine eligibility for inclusion in this review.

Next, a hand search of seven select journals was conducted to identify articles that may have been left out of the electronic searches, including *Exceptional Children, Journal of Learning Disabilities, Journal for Research in Mathematics Education, Learning Disabilities Research & Practice, Remedial and Special Education, Research in Developmental Disabilities, and School Psychology Quarterly.* Additionally, three recently published mathematics intervention review articles or meta-analyses were searched (Baker, Gersten, & Lee, 2002; Burns, Coddling, Boice, & Lukito, 2010; Coddling, Hilt-Panahon, Panahon, & Benson, 2009) to ensure as many studies as possible were located for potential inclusion in this literature review. Reference sections of articles that met the search criteria were also considered to locate studies that again may have been missed from the electronic search. Eleven studies met inclusion criteria for this review. The majority of the 87 studies were excluded for the following reasons: target interventions included word problems, decimals and fractions, problem solving, homework, or coaching rather than flashcards; flashcards were used but not in a direct instruction manner, i.e., cover, copy, compare; studies were not about math; participants were not students with disabilities; articles were meta-analyses, secondary data analyses, or

commentaries that did not specifically examine the use of flashcards; or studies did not consist of at least one dependent variable that measured academic outcomes, such as digits correct per minute, percentage of correct responses, or accuracy of problems completed.

Results

Of the 11 studies that met inclusion criteria for this review, ten employed some variation of a single-case design (e.g., alternating treatments, multiple probe, or multiple baseline), while one utilized a pre-post design (Beirne-Smith, 1991). A total of 53 participants identified as having a disability (LD = 31; LD and Attention Deficit Hyperactivity Disorder, ADHD = 1; ADHD and Developmental Disabilities, DD = 2; DD = 2; Oppositional Defiant Disorder, ODD and ADHD = 2) or as demonstrating skills deficits in mathematics ($n = 15$) were included in these studies. The majority of studies targeted students in grades 2 – 4, while students in grades 5, 6, 7, 9, 10, and 11 were also included. A majority of studies ($n = 8$) focused on multiplication as the operation for which flashcards were employed, while one study targeted all four operations, one study targeted both multiplication and division, and one study targeted addition. Time of intervention per session typically ranged from 10 minutes to 30 minutes, although not all studies reported this information. Dependent variables used to demonstrate the effectiveness of flashcard instruction to teach basic math facts included percentage of correct responses, percentage of digits correct, number of sessions to reach mastery, number of correct responses, number of facts correct, problems correct, problems correct per minute, errors per minute, digits correct per minute, and opportunities to respond.

Types of Flashcard Interventions

Three different types of flashcards interventions (direct instruction procedures, ratio/incremental rehearsal techniques, and peer tutoring with flashcards) for teaching basic math

facts to students with disabilities or demonstrating skills deficits in mathematics emerged. Direct instruction procedures included the model, lead, test, retest, and immediate feedback format (Brasch, Williams, & McLaughlin, 2008; Delli Sante, McLaughlin, & Weber, 2001; Glover, McLaughlin, Derby, & Gower, 2010; Hayter, Scott, McLaughlin, & Weber, 2007; Wilson, Majsterek, & Simmons, 1996). Ratio and incremental rehearsal techniques included use of specific ratios of known and unknown facts to teach basic math facts (Burns, 2005; Coddling, Archer, & Connell, 2010; Cooke, Guzaukas, Pressley, & Kerr, 1993). Studies that used peer tutoring examined rapid re-presentation of facts or a counting-on approach versus a rote-memorization approach (Beirne-Smith, 1991; Rhymer, Dittmer, Skinner, & Jackson, 2000; Van Houten & Rolider, 1989). All three types of interventions included various components of effective instruction, such as opportunities to respond, cumulative review, and immediate feedback, in one manner or another. A summary of included studies and key findings is presented in Table 1.

Direct instruction procedures. In the five studies that incorporated direct instruction procedures including model, lead, test, retest, and immediate feedback, all five measured number of correct responses by examining total number of facts (responses) correct or problems correct per minute, and in two studies (Delli Sante et al., 2001; Hayter et al., 2007), errors per minute. All five studies used multiplication as the operation for which flashcards were implemented; one study (Glover et al., 2010) also used division for one participant. Participants included students identified with LD, DD and ADHD, DD, ADHD and ODD, or LD and ADHD.

Wilson et al. (1996) used a single-case, alternating treatments design to investigate the effects of computer-assisted instruction (CAI) versus teacher-directed instruction (TDI) on the multiplication performance of students with LD. The TDI phase included the use of flashcards

in a direct instruction manner, e.g., “My turn. Two times nine equals eighteen. Say it with me. Two times nine equals eighteen. Say it by yourself.” Teachers immediately corrected errors by implementing the teacher alone, teacher and student together, and student alone sequence until correct answers were provided. Teachers praised correct responses immediately and provided verbal feedback for self-corrections or slow responses. The CAI phase consisted of the Math Blaster (Eckert & Davidson, 1987) program, where students engaged in “Look & Learn,” “Build Your Skill,” and an arcade-style game comprised of practice on facts previously mastered. All four students mastered more facts in the TDI condition as measured by the number of facts mastered, had more opportunities to respond in the TDI condition, and had a higher mastery rate in the TDI condition as measured by percentage of correct responses, as compared to the CAI condition.

Delli Sante et al. (2001) utilized a multiple baseline design across two participants with DD and ADHD to evaluate the effectiveness of using a direct instruction flashcard intervention to increase multiplication facts mastery and decrease number of errors demonstrated by participants. During every session 15 flashcards were presented three times each, using the model, lead, test, and retest format. Once the participant could name a fact in three consecutive sessions that fact was removed from the group of 15 and was replaced by a fact that was incorrect during baseline. Both students displayed increases in the number of problems correct per minute (pc/m) and decreases in the number of errors per minute (e/m) once the direct instruction flashcards intervention was implemented. Student 1 improved from an average of 4.75 pc/m during baseline to an average of 30.8 pc/m during intervention; Student 1 decreased from an average of 12.25 e/m during baseline to an average of 2.06 e/m during intervention. Student 2 improved from an average of 4 pc/m during baseline to an average of 25.77 pc/m

during intervention; Student 2 decreased from an average of 7.6 e/m during baseline to an average of 2.0 e/m during intervention.

Hayter et al. (2007) also utilized a multiple baseline design across two participants with DD to measure effectiveness of a direct instruction flashcard intervention on both correct responses and number of errors per minute. All basic multiplication facts for 4's were used in this study and were presented via flashcards and printed worksheets. Instruction was delivered to both participants at the same time using a direct instruction format, e.g., "After I snap you will say the whole math fact, 'Zero times four is zero.'" The first author immediately corrected any errors, said the problem and answers herself, said the problem and answers with both students, and expected the students to say the problem and answer without her help. Both students demonstrated increases in pc/m and decreases in e/m when the flashcard system was implemented. Student 1 improved from a median of 5 pc/m during baseline to a median of 22.5 pc/m during intervention; Student 1 decreased from a median of 2.3 e/m during baseline to a median of 0.09 e/m during intervention. Student 2 improved from a median of 8 pc/m during baseline to a median of 15.4 pc/m during baseline; Student 2 decreased from a median of 6.5 e/m during baseline to a median of 3.7 e/m during intervention.

Brasch et al. (2008) used a multiple baseline across three different sets of eight multiplication facts with two students with ADHD and ODD to investigate a direct instruction flashcard procedure on the mastery of multiplication facts. A second dependent variable was the number of sessions it took the students to reach mastery, as defined by telling the correct answer within two seconds after the problem was presented. If a student responded incorrectly or took longer than two seconds, the first author immediately modeled the problem and correct answer, said the problem and correct answer with the student, and asked the student to repeat the problem

and correct answer. Both students exhibited increases in correct responding during flashcard intervention. Student 1 improved from a mean of less than 1 problem correct (pc) during baseline to a mean of 7.8 pc during intervention with an average of 3.5 sessions to reach mastery. Student 2 improved from a mean of 2.5 pc during baseline to a mean of 7.5 pc during intervention with an average of 4 sessions to reach mastery.

Glover et al. (2010) employed a multiple baseline design across two participants with LD, one of whom also had ADHD, to increase corrects rate and decrease errors rate for see/write digits in multiplication facts (Student 1) and division facts (Student 2) using a direct instruction flashcard system and a back three strategy to address errors. Contingent praise consisting of high fives and other compliments was used in conjunction with the flashcard intervention. The researchers modeled saying the whole problem and answer and expected a response within two seconds. If an incorrect answer was provided or a student had a delay of longer than two seconds, the researchers would provide a model, lead, test procedure to review the fact. Incorrects were placed three cards back in the deck from the top of the deck, thus ensuring students would have almost immediate opportunities to revisit the fact. Both participants improved their mastery of multiplication or division facts as demonstrated by increases in number of correct responses (n/cr). Student 1 improved from a mean baseline of 3.25 n/cr during baseline to a mean of 7.0 n/cr during intervention. Student 2 improved from a mean of 2.59 n/cr during baseline to a mean of 8.0 n/cr during intervention.

Ratio and incremental rehearsal techniques. In the three studies that incorporated ratio and incremental rehearsal techniques using specific ratios of known and unknown facts to teach basic math facts, all three measured number of correct responses by examining digits correct per minute (dcpm). All three studies used multiplication as the operation for which

flashcards were implemented. Participants included students identified with LD or skills deficits in mathematics, specifically with basic multiplication facts.

Cooke et al. (1992) used an alternating treatments design to compare the effects of two ratios (30/70% new/review items to 100% new items) on multiplication fact fluency of three students with LD. After presentation of flashcards, an incorrect response or delay of greater than two seconds resulted in the teacher stating the correct answer, asking the student to repeat the answer, repeating those steps two more times, and presenting the missed fact after each of the next four facts. Once a student could answer a fact correctly and within two seconds over a minimum of three consecutive sessions, it was considered mastered and moved to the 70% review items category for future sessions. In the 100% new items condition, all facts were selected from the group of unknown facts; once a fact was mastered it was replaced with a different unknown fact. Higher rates of fluency were evident for all three students under the 30/70% new/review items condition as opposed to the 100% new items. All students improved from baseline of under 10 dcpm to rates ranging from 24 dcpm to 65 dcpm following implementation of flashcards.

Burns (2005) implemented a multiple baseline single-case design across three students with LD to evaluate the effectiveness of incremental rehearsal to increase fluency in multiplication facts. Similar to Cooke et al. (1992), facts were considered as known if the correct answer was provided within two seconds; incorrect answers or answers with a greater delay than two seconds were considered as unknown and were taught to students using a model, lead, test format. Incremental rehearsal techniques, in which a “gradually increasing ratio of known to unknown items reaches, at the final stage of implementation, 90% to 10%,” (Burns, 2005, p. 238) were used to instruct students on unknown facts. The number of flashcards per

session was held constant at 10 and unknown facts were rehearsed following a specific sequence of presentation. All three students demonstrated increases in dcpm from baseline phase to intervention phase, and all three displayed immediate changes in level and trend once intervention began. Student 1 improved from a median baseline point of 3 dcpm during baseline to a median of 15 dcpm during intervention. Student 2 improved from a median baseline point of 8 dcpm during baseline to a median of 25 dcpm during intervention. Student 3 improved from a median baseline point of 11 dcpm during baseline to a median of 27 dcpm during intervention.

Codding et al. (2010) applied a multiple-probe design across multiplication facts with one student displaying skills deficits in mathematics to systematically replicate Burns' (2005) study using incremental rehearsal to improve multiplication skills. Codding used 10 flashcard facts per session, one unknown and nine known. The unknown fact and answer was presented to the student by the interventionist, the student was asked to restate the problem and answer, and the sequence was presented again. The same procedure was followed with the remaining known facts. Across the three sets of multiplication facts, the student's baseline performance was in the frustrational range (Burns, VanDerHeyden, & Jiban, 2006), with mean dcpm at 3.8, 21.1, and 22.8 for Sets A, B, and C respectively. Following intervention, mean dcpm was 30.0, 48.7, and 44.6 for Sets A, B, and C respectively. Across the three sets, performance reached mastery level criteria of greater than 49 dcpm as defined by Burns et al. (2006).

Peer tutoring with flashcards. In the three studies that incorporated peer tutoring with flashcards, problems correct and percentage of correct responses were used to determine effectiveness of flashcard interventions with 2nd-4th grade students. One study used multiplication flashcards, one used addition, and one study used all four operations dependent on the particular student. Students were identified with LD or skills deficits in mathematics.

Van Houten and Rolider (1989) implemented an alternating treatments design with 10 students demonstrating skills deficits in mathematics specifically with the learning of addition, subtraction, multiplication, or division basic facts. The authors compared sequential presentation of unknown facts (i.e., all eight facts were presented at one time, shuffled, then presented seven more times) with rapid re-presentation of missed facts (i.e., re-presenting missed facts immediately after presentation of the next fact), presented by same-age tutors without skills deficits. A lower number of sessions to reach mastery were evident for students receiving rapid re-presentation of facts; mean number of sessions to mastery was 7.4, while mean number of sessions to mastery was 9.5 for sequential presentation of facts. Percentage of correct responses increased more for all students under the rapid re-presentation condition than under the sequential presentation condition.

Beirne-Smith (1991) implemented a pre-post-test design to investigate the use of peer tutoring with addition flashcards for 20 students with LD. Students were placed into either a counting-on approach for addition facts (Method A), a rote memorization flashcard approach (Method B), or a no-treatment control condition (Method C). In Method A, tutors followed a five-step procedure: a) rule-stating, "Each time the addend increases by one the sum increases by one. Say it with me."; b) demonstration, e.g., "My turn, $2 + 4 = 6$."; c) unison responding, e.g., "Say it with me."; d) individual turns, e.g., "Say it by yourself."; e) testing, "Say it again." In Method B, tutors modeled facts by saying, "My turn, $2 + 6 = 8$," then asking the students, "Your turn, $2 + 6 =$ how many?" Mean number of problems correct (pc) before implementation of flashcards (pretest) were 23.8 pc for students assigned to Method A, 23.1 pc for students assigned to Method B, and 22.9 pc for students assigned to Method C. After implementation of flashcards (posttest), mean number of problems correct were 40.1 pc for students assigned to

Method A, 38.1 pc for students assigned to Method B, and 26.3 pc for students assigned to Method C. Thus, both versions of peer tutoring with flashcards were more effective than no-treatment in the learning of basic math facts for students with LD.

Rhymer et al. (2000) used a single-case, alternating treatments design to examine the effectiveness of an intervention that combined explicit timings, peer tutoring, positive-practice overcorrection, and immediate performance feedback on mathematics fluency for elementary students with skills deficits in mathematics. All students alternated between the role of tutor, tutee, and control conditions. Flashcards were used by tutors in a direct instruction manner, with specific focus on positive-practice overcorrection and immediate performance feedback, e.g., “That is incorrect; the answer to four times two is eight.” The tutee would then write the whole problem and correct answer three times on a piece of paper. All four students demonstrated an increase in fluency as measured by problems correct per minute when performance feedback was provided. When serving as both tutor (delivering feedback) and tutee (computation responding), three of the four participants showed increases in rates of accurate responding, thereby providing support for interventions that combine both accuracy and fluency components.

Fidelity of Treatment

Following Horner et al.’s (2005) recommendations of quality indicators to identify effectiveness of interventions, eight of the 11 included studies measured fidelity of implementation of treatment or treatment integrity. Fidelity of treatment information is necessary to provide information for future systematic replication of existing studies (Misquitta, 2011). In this review, studies that reported they measured treatment fidelity also described procedures used to do this, such as using procedure checklists, having a second researcher simultaneously recording data at the same time as the first researcher, scoring by an additional

person, and having a second observer taking data on both assessment and intervention procedures (Brasch et al., 2008; Burns, 2005; Coddling et al., 2010; Cooke et al., 1993; Delli Sante et al., 2001; Glover et al., 2010; Hayter et al., 2007; Rhymer et al., 2000).

Settings and Length of Interventions

Settings in which studies were conducted were generally similar. Of included studies, six took place in special education resource or self-contained classrooms (Burns, 2005; Cooke et al., 1993; Delli Sante et al., 2001; Glover et al., 2010; Hayter et al., 2007; Wilson et al., 1996), and five took place in small, empty classrooms or computer labs (Beirne-Smith, 1991; Brasch et al., 2008; Coddling et al., 2010; Rhymer et al., 2000; Van Houten & Rolider, 1989). Most studies reported time of implementation as between 10 minutes per session to 30 minutes per session; Cooke et al. (1993), Delli Sante et al. (2001), and Glover et al. (2010) did not report for how long each session was implemented. The total number of sessions for which flashcards interventions were implemented ranged from seven sessions to 30 sessions; Cooke et al. did not report duration for their study. A number of studies reported implementation occurred between two and four times per week (Burns, 2005; Coddling et al., 2010; Delli Sante et al., 2001; Hayter et al., 2007), other studies reported daily implantation (Beirne-Smith, 1991; Brasch et al., 2008), while the remaining five studies did not mention for how many days per week intervention took place.

Types of Disabilities

Of included studies, all included at least one subject who was identified as having some type of disability or demonstrated skills deficits in mathematics. Van Houten and Rolider (1989) reported 10 students with skills deficits in mathematics, Rhymer et al. (2000) reported four students with skills deficits in mathematics, and Coddling et al. (2010) reported 1 student with

skills deficits in mathematics, for a total of 15 students with skills deficits in mathematics. Beirne-Smith (1991), Cooke et al. (1993), Wilson et al. (1996), Burns (2005), and Glover et al. (2010) reported a total of 31 participants identified with learning disabilities. Delli Sante et al. (2001) reported both participants as identified with DD and ADHD, Brasch et al. (2008) reported both participants as identified with ADHD and ODD, Glover et al. (2010) reported one participant as identified with both LD and ADHD, and Hayter et al. (2007) reported both participants as identified with DD.

Overall findings from this review about specific components of flashcard instruction as a method to increase mathematics performance of students with disabilities or demonstrating skills deficits in mathematics indicated favorable results and increases in students' math performance when flashcard interventions are employed. Flashcards were implemented: in a direct instruction manner using model-lead-test formats, by using incremental rehearsal techniques with specific ratios of known and unknown facts, or by incorporating peer-tutoring and flashcards together. Positive outcomes included improvements in digits correct per minute, percentage of correct problems, number of basic math facts mastered, and decreases in error per minute. Positive outcomes were identified for students with LD, DD, ADHD, ODD, and those who demonstrated skills deficits in mathematics.

Discussion

Effective instruction in basic math facts is especially critical to students with SLD or those demonstrating skills deficits in mathematics, as proficiency with 390 basic facts is now expected as early as the end of grade 3 (CCSSI, 2010; NMAP, 2008). As mastery of basic math facts is considered an essential foundational skill for more complex mathematic tasks such as algebra and fractions (Coddling et al., 2009), identifying available interventions that may improve

students' mathematics performance is of utmost importance. The purpose of this article was to identify and summarize existing literature on interventions that utilized flashcards to teach basic math facts to students with SLD or demonstrating skills deficits in mathematics.

Recent reviews in the area of mathematics have emphasized the need for frequent and multiple opportunities for students to practice discrete skills. Implementation of flashcards is one strategy that has been associated with more frequent opportunities to practice activities such as sight words, word recognition, and letter sounds. Similarly, support for the use of flashcards as a simple intensity intervention that could be implemented with skills in mathematics as a way to provide more practice opportunities, specifically for students with SLD in mathematics, was noted by a number of researchers. Flashcards are easy to implement, do not require a great deal of resources, and can be implemented quickly and easily by teachers, paraprofessionals, parents, or even peers (Nist & Joseph, 2008). Three primary components of flashcard instruction became apparent from this review.

Types of Flashcard Interventions

Implementation of flashcards in a direct instruction manner following the model, lead, test, retest, and immediate feedback format occurred in five studies and was found to improve number of correct problems and decrease number of errors for students with various disabilities in the areas of multiplication and division (Brasch et al., 2008; Delli Sante et al., 2001; Glover et al. 2010; Hayter et al., 2007; Wilson et al., 1996). Studies utilizing flashcards in this manner included use of praise for correct responses, corrective and immediate feedback in the case of errors, and multiple opportunities for students to practice specific facts. Sequence of teaching facts and correcting errors included: teacher alone, teacher and student together, and student alone until facts were considered mastered. Mathematics performance of all participants who

received flashcard instruction in this manner improved in terms of percentage of correct responses or number of problems correct; fewer numbers of sessions to reach mastery and decreases in number of errors were also positive results associated with this type of instruction.

Implementation of flashcards in a manner that incorporated ratio and incremental rehearsal techniques including use of specific ratios of known and unknown facts to teach basic math facts took place in three studies (Burns, 2005; Coddling et al., 2010; Cooke et al., 1993). Studies using flashcards in this manner used ratios of 30/70% new/review items to 100% new items or 90% known to 10% unknown as ways to teach basic math facts to students. Mathematics of all participants who received flashcard instruction in this manner improved in terms of digits correct per minute.

Implementation of flashcards in a manner that included peer tutoring and examined rapid re-presentation of facts or a counting-on approach versus a rote-memorization approach occurred in three studies (Beirne-Smith, 1991; Rhymer et al, 2000; Van Houten & Rolider, 1989). Same-age tutors were used to implement flashcards with students with SLD or demonstrating skills deficits in mathematics; in two of the studies, flashcards were implemented by tutors in a direct instruction manner following the model, lead, test format. Percentage of facts correct or number of correct responses increased for all but one student in these studies after flashcards were implemented.

Settings and Types of Disabilities

Settings in which flashcards were implemented were generally similar, using either special education resource or self-contained classrooms or small, empty classrooms or computer labs. The setting did not appear to have any effect on the implementation of flashcards; therefore, it appears as though flashcards could be implemented in any type of setting and still

lead to the same positive outcomes. Most studies reported time of implementation as between 10 minutes per session to 30 minutes per session; meaning the implementation of flashcards should not replace typical mathematics instruction, rather it should serve as a supplement to traditional instruction. Participants in included studies were identified with LD, LD and ADHD, ADHD and DD, DD, and ODD and ADHD or demonstrating skills deficits. Flashcard use was effective for all but one participant in the total pool of participants, which demonstrates flashcards could be used with students with a variety of disabilities, in a variety of settings.

Implications for Practice

Flashcard instruction that was implemented in direct instruction, incremental rehearsal, and/or peer tutoring manners led to improvements in students' mathematics performance in terms of digits correct per minute, percentage of correct responses, and number of math facts mastered, to name just a few areas where increases were evident. Past reviews in effective instruction in mathematics have all emphasized the need for multiple and frequent opportunities for students to practice specific skills (e.g., Burns et al., 2010; Coddling et al., 2009; Coddling et al., 2011; Mercer & Miller, 1992). Repeated practice has been associated with developing automaticity, fluency, and rapid recognition with discrete skills (Volpe et al., 2011), such as sight words and basic math facts. This review demonstrated implementation of flashcards is one strategy that can provide students with more frequent opportunities to practice basic math facts. Typically, students are expected to move on to new facts before mastery has occurred and are often only provided "instruction" through mad minute assessments (Crawford, 2003), without opportunities to practice new facts. Implementing flashcard instruction is one way to ensure students, especially those with SLD or demonstrating skills deficits in mathematics, receive actual instruction in basic math facts and are given opportunities to practice before moving on.

Flashcards are easy to implement, do not require a great deal of physical or monetary resources, and can be implemented by teachers, paraprofessionals, parents, or even peers (Nist & Joseph, 2008).

Limitations and Future Directions

A main limitation of this review is the small number of studies that were found to meet inclusion criteria in this review, which indicates a continued need for high-quality research that is conducted with basic math facts interventions. Also, of the 11 included studies, only four reported maintenance data for participants; one of the four studies only reported maintenance data for one day post-intervention. Only one study reported generalization data and three failed to report any information about fidelity of treatment. Similarly, three studies did not report time of intervention per session, while five studies failed to report for how many days per week intervention occurred. Given the small number of included studies and studies failing to report maintenance data, generalization data, fidelity of treatment data, time per session, or number of days per week of intervention, there continues to exist a growing need for high-quality research that includes such information.

Conclusions

Findings from this review suggest use of flashcard instruction is one effective intervention to improve mathematics performance of students with SLD or demonstrating skills deficits in mathematics. Flashcards provide students with increased opportunities to respond and considerable amounts of practice with basic math facts. If implemented in a direct instruction manner, flashcards can include demonstration or modeling of specific skills, guided practice with immediate feedback and prompts, and independent practice coupled with feedback, all of which have been demonstrated as components of effective instruction, particularly for students with

SLD or demonstrating skills deficits in mathematics. Teachers, especially those who teach students with disabilities, should make deliberate efforts to include more opportunities for all students to respond and remain actively engaged in mathematics instruction. Even though the number of studies reviewed is small, common themes which emerged could provide a guide to practitioners on effective instructional procedures with basic math facts, such as using flashcards in direct instruction, ratio or incremental rehearsal, or peer tutoring approaches. Given the importance of basic math facts as related to future and more complex mathematics skills as students progress through school, more research is needed to identify effective instructional strategies in this critical area of mathematics.

References

Note: References marked with an asterisk denote studies included in this review.

- Baker, S., Gersten, R., & Lee, D.-S. (2002). A synthesis of empirical research on teaching mathematics to low-achieving students. *The Elementary School Journal*, *103*(1), 51-73.
- * Beirne-Smith, M. (1991). Peer tutoring in arithmetic for children with learning disabilities. *Exceptional Children*, *57*(4), 330-337.
- * Brasch, T.L., Williams, R.L., & McLaughlin, T.F. (2008). The effects of a direct instruction flashcard system on multiplication fact mastery by two high school students with ADHD and ODD. *Child & Family Behavior Therapy*, *30*(1), 51-59.
- * Burns, M. K. (2005). Using incremental rehearsal to increase fluency of single-digit multiplication facts with children identified as learning disabled in mathematics computation. *Education and Treatment of Children*, *28*(3), 237-249.
- Burns, M.K., VanDerHeyden, A.M., & Jiban, C. (2006). Assessing the instructional level for mathematics computation problems: A comparison of methods. *School Psychology Review*, *35*, 401-418.
- Burns, M.K., Coddling, R.S., Boice, C.H., & Lukito, G. (2010). Meta-analysis of acquisition and fluency math interventions with instructional and frustration level skills: Evidence for a skill-by-treatment interaction. *School Psychology Review*, *39*(1), 69-83.
- Cawley, J.F., & Miller, J.H. (1989). Cross-sectional comparisons of the mathematical performance of children with learning disabilities: Are we on the right track toward comprehensive programming? *Journal of Learning Disabilities*, *23*, 250-254, 259.
- Coddling, R.S., Eckert, T.L., Fanning, E., Shiyko, M., & Solomon, E. (2007). Comparing mathematics interventions: The effects of cover-copy-compare alone and combined with

- performance feedback on digits correct and incorrect. *Journal of Behavioral Education*, 16(2), 125-141.
- Codding, R.S., Hilt-Panahon, A., Panahon, C.J., & Benson, J.L. (2009). Addressing mathematics computation problems: A review of simple and moderate intensity interventions. *Education and Treatment of Children*, 32(2), 279-312.
- * Codding, R.S., Archer, J., & Connell, J. (2010). A systematic replication and extension of using incremental rehearsal to improve multiplication skills: An investigation of generalization. *Journal of Behavioral Education*, 19, 93-105.
- Codding, R.S., Burns, M.K., & Lukito, G. (2011). Meta-analysis of mathematics basic-fact fluency interventions: A component analysis. *Learning Disabilities Research & Practice*, 26(1), 36-47.
- Common Core State Standards Initiative. (2010). *Common core state standards for mathematics: Grades K – 8*.
- * Cooke, N.L., Guzaukas, R., Pressley, J.S., & Kerr, K. (1993). Effects of using a ratio of new items to review items during drill and practice: Three experiments. *Education and Treatment of Children*, 16, 213-234.
- Crawford, D.B. (2003). *How to Successfully Teach Math Facts to All Students: Workshop Manual*. Eau Claire, WI: Otter Creek Institute.
- Daly, E.J., Martens, B.K., Barnett, D., Witt, J.C., & Olson, S.C. (2007). Varying intervention delivery in response to intervention: Confronting and resolving challenges with measurement instruction and intensity. *School Psychology Review*, 36, 562-581.
- * Delli Sante, A., McLaughlin, T.F., & Weber, K.P. (2001). The use and evaluation of a direct instruction flash card strategy on multiplication math facts mastery with two students

- with developmental disabilities and attention deficit hyperactivity disorder. *Journal of Precision Teaching and Celeration*, 2, 68-75.
- Doabler, C.T., Cary, M.S., Jungjohann, K., Clarke, B., Fien, H., Baker, S., . . . Chard, D. (2012). Enhancing core mathematics instruction for students at risk for mathematics disabilities. *Teaching Exceptional Children*, 44(4), 48-57.
- Eckert, R., & Davidson, J. (1987). *Math blaster plus* [Computer software]. Torrance, CA: Davidson & Associates.
- Erbey, R., McLaughlin, T.F., Derby, K.M., & Everson, M. (2011). The effects of using flashcards with reading racetrack to teach letter sounds, sight words, and math facts to elementary students with learning disabilities. *International Electronic Journal of Elementary Education*, 3(3), 213-226.
- Geary, D.C. (2004). Mathematics and learning disabilities. *Journal of Learning Disabilities*, 37(1), 4-15.
- Gersten, R., Beckmann, S., Clarke, B., Foegen, A., Marsh, L., Star, J. R., & Witzel, B. (2009). *Assisting students struggling with mathematics: Response to Intervention (RTI) for elementary and middle schools* (NCEE 2009-4060). Washington, DC: National Center for Education Evaluation and Regional Assistance, Institute of Education Sciences, U.S. Department of Education. Retrieved from:
http://ies.ed.gov/ncee/wwc/pdf/practice_guides/rti_math_pg_042109.pdf
- * Glover, P., McLaughlin, T., Derby, K.M., & Gower, J. (2010). Using a direct instruction flashcard system with two students with learning disabilities. *Electronic Journal of Research in Educational Psychology*, 8(2), 457-472.

- * Hayter, S., Scott, E., McLaughlin, T.F., & Weber, K.P. (2007). The use of a modified direct instruction flashcard system with two high school students with developmental disabilities. *Journal of Developmental and Physical Disabilities, 19*, 409-415.
- Heron, T.E., Heward, W.L., Cooke, N.L., & Hill, A.S. (1983). Evaluation of a classwide peer tutoring system: First graders teach each other sight words. *Education and Treatment of Children, 6*, 137-152.
- Horner, R.H., Carr, E.G., Halle, J., McGee, G., Odom, S., & Wolery, M. (2005). The use of single-subject research to identify evidence-based practice in special education. *Exceptional Children, 71*(2), 165-179.
- Jordan, N.C., Kaplan, D., & Hanich, L.B. (2002). Achievement growth in children with learning difficulties in mathematics: Findings of a two-year longitudinal study. *Journal of Educational Psychology, 94*, 586-597.
- Kroesbergen, E.H., & Van Luit, J.E. (2003). Mathematics interventions for children with special educational needs: A meta-analysis. *Remedial and Special Education, 24*(2), 97-114.
- McClelland, M.M., Acock, A.C., & Morrison, F.J. (2006). The impact of kindergarten learning related skills on academic trajectories at the end of elementary school. *Early Childhood Research Quarterly, 21*, 471-490.
- Mercer, C.D., & Miller, S.P. (1992). Teaching students with learning problems in math to acquire, understand, and apply basic math facts. *Remedial and Special Education, 13*(3), 19-35.
- Miller, S., & Mercer, C. (1997). Educational aspects of mathematics disabilities. *Journal of Learning Disabilities, 30*, 47-56.

- Misquitta, R. (2011). A review of the literature: Fraction instruction for struggling learners in mathematics. *Learning Disabilities Research & Practice, 26*(2), 109-119.
- National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: Author.
- National Mathematics Advisory Panel. (2008). *Foundations for Success: The Final Report of the National Mathematics Advisory Panel*, U.S. Department of Education: Washington, DC.
- Nist, L., & Joseph, L.M. (2008). Effectiveness and efficiency of flashcard drill instructional methods on urban first-graders' word recognition, acquisition, maintenance, and generalization. *School Psychology Review, 37*, 294-308.
- * Rhymer, K.N., Dittmer, K.I., Skinner, C.H., Jackson, B. (2000). Effectiveness of a multi-component treatment for improving mathematics fluency. *School Psychology Quarterly, 15*(1), 40-51.
- Rosenshine, B., & Stevens, R. (1986). Teaching functions. In M.C. Wittrock (Ed.), *Handbook of research on teaching* (3rd ed., pp. 376-391). New York: Macmillan.
- Stein, M., Silbert, J., & Carnine, D. (2006). *Designing effective mathematics instruction: A direct instruction approach* (4th ed.). Upper Saddle River, NJ: Pearson Education.
- * Van Houten, R., & Rolider, A. (1989). An analysis of several variables influencing the efficacy of flash card instruction. *Journal of Applied Behavior Analysis, 22*, 111-118.
- Volpe, R.J., Mule, C.M., Briesch, A.M., Joseph, L.M., & Burns, M.K. (2011). A comparison of two flashcard drill methods targeting word recognition. *Journal of Behavioral Education, 20*, 117-137.

* Wilson, R., Majsterek, D., & Simmons, D. (1996). The effects of computer-assisted versus teacher-directed instruction on the multiplication performance of elementary students with learning disabilities. *Journal of Learning Disabilities, 29*(4), 382-390.

Zentall, S.S. (1990). Fact-retrieval automatization and math problem solving by learning disabled, attention-disordered, and normal adolescents. *Journal of Educational Psychology, 82*(4), 856-865.

Table 1

Effectiveness of Basic Math Facts Interventions for Students with Disabilities or Demonstrating Skills Deficits

Citation	Number of participants; Grade level	Disability	Operation(s)	Intervention(s); Dependent Variable(s)	Results
Van Houten & Rolider (1989)	10; 2 nd and 4 th	Skills deficits in math facts	Addition, subtraction, multiplication, division	Rapid re-presentation of flash cards; 12-minute sessions, 13 total sessions, percentage of correct responses	Percentage of correct responses increased more under rapid re-presentation condition than sequential presentation condition
Beirne-Smith (1991)	20; 2 nd – 4 th	LD	Addition	Peer tutors taught a counting-on approach vs. rote-memorization using flashcards vs. a no-treatment approach; Problems correct (pc), untimed	Significant differences found for both peer tutoring using flashcards and counting-on approach over no-treatment group. Group using flashcards increased in pc from 23.1 to 38.1
Cooke, Guzaukas, Pressley, & Kerr (1993)	3; 3 rd , 4 th , and 6 th	LD	Multiplication	Flashcards with immediate feedback; digits correct per minute (dcpm)	All students made gains in fluency (Range =24dcpm-65dcpm; baseline = <10dcpm)
Wilson, Majsterek, & Simmons (1996)	4; 3 rd and 4 th	LD	Multiplication	Computer-assisted (CAI) versus teacher-directed instruction (TDI) using flashcards; opportunities to respond and mastery rate	All mastered more facts in TDI condition, students had more opportunities to respond in TDI and had higher mastery rate than CAI

Rhymer, Dittmer, Skinner, & Jackson (2000)	4; 4 th	Skills deficits in math	Multiplication	Multi-component treatment – timings, peer tutoring, positive-practice overcorrection, and performance feedback; problems correct per minute (pc/m)	Serving as both tutor and tutee led to greater pc/m for 3/4 students; all 4 increased in fluency when performance feedback was added
Delli Sante, McLaughlin, & Weber (2001)	2; 6 th	DD and ADHD	Multiplication	Direct instruction flashcards, 40 days; problems correct/minute (pc/m) and errors/minute (e/m)	Direct instruction flashcards increased pc/m and decreased e/m for both participants
Burns (2005)	3; 3 rd	LD	Multiplication	Incremental rehearsal using flashcards, 2 times/week for 15 weeks; weekly CBM fluency probes scored as dc/m	All 3 students demonstrated increases in dc/m from baseline to intervention and 100% PND, immediate change in level and trend once intervention began
Hayter, Scott, McLaughlin, & Weber (2007)	2; 10 th and 11 th	DD	Multiplication	Modified direct instruction flashcard system, 10 minutes per session for 14 days; number of problems correct per minute (pc/m) and errors per minute	Both students demonstrated increases in pc/m and decreases in e/m when flashcard system was used. Pc/m increased from 5 to 22.5 and 8 to 15.4 for the two students
Brasch, Williams, &	2; 9 th and 11 th	ADHD and ODD	Multiplication	Direct Instruction Flashcard System for	Large increases were demonstrated by both

McLaughlin (2008)				fact mastery was implemented; 15 mins/day for 14 days. Number of correct responses to multiplication flashcards and number of sessions to mastery.	participants in number of correct responses. Student 1 increased from 0 correct responses to 7 with flashcards; Student 2 increased from about 2 correct responses to about 7 with flashcards. Student 1 averaged 3.5 sessions to mastery; Student 2 averaged 4 sessions to mastery
Codding, Archer, & Connell (2010)	1; 7 th	Skills deficits in math (specifically, basic multiplication facts)	Multiplication	Incremental rehearsal based on Burns' (2005) study using flashcards, Twice weekly across 12 weeks for about 20 minutes each session; digits correct per minute (dcpm) and percentage of digits computed correctly across problem sets	Baseline performance fell in frustration range; performance after intervention met mastery level criteria (>49dcpm) and was maintained across sets. Percentage increased from 61% accuracy to 100% accuracy
Glover, McLaughlin, Derby, & Gower (2010)	2; 5 th	LD, 1 also had ADHD	Multiplication for one student, division for one student	Direct instruction flashcard procedure was implemented across 3 sets of facts; study was conducted in resource room; Number of corrects	Each participant improved their mastery of math facts as demonstrated by increases in number of corrects for each participant.

Note. ADHD = attention deficit hyperactivity disorder; DD = developmental disabilities; LD = learning disabilities; ODD = Oppositional Defiant Disorder.

Vita

KAREN M. FRIES

The Pennsylvania State University
122 CEDAR
University Park, PA 16802
843-437-0810
kmf5111@psu.edu

Education

PhD		Pennsylvania State University (anticipated date Aug. 2013)	Special Education
MS	2000	University of New York at Albany	Special Education
BS	1995	George Mason University	Psychology

Licensures and Certifications

South Carolina, Special Education, Generic and ED, PreK-12th grade
New York, Special Education, Generic, PreK-12th grade

Publications

Fries, K.M., & Riccomini, P.J. (2012). Effects of PALS-math on mathematics performance of students with disabilities. *Focus on Inclusive Education*, 10(1), 1-4.

Correa, V.C., Baughan, C.C., **Fries, K.M.**, Thompson, M.P., Algozzine, R. F. (In press). Evaluation of the effects of Read it Once Again across two groups of students. *NHSA Dialog: A Research-to-Practice Journal for the Early Childhood Field*.

Fries, K.M., & Riccomini P.J. (In preparation). Effectiveness of mastering math facts on fourth-grade students with or at-risk for specific learning disabilities in mathematics.

Recent Presentations

Fries, K.M. (2013, April). *Effectiveness of Math Facts Practice on Fourth-Grade Students with Learning Disabilities in Mathematics*. Presentation made at the 2013 Annual Convention and Expo for Exceptional Children. San Antonio, TX. Refereed Conference Presentation.

Fries, K.M. (2013, April). *Effects of PALS-Math on Mathematics Performance of Students with Disabilities: A Literature Review*. Presentation made at the 2013 Annual Convention and Expo for Exceptional Children. San Antonio, TX. Poster presentation.

Professional Experience

2010-2013 The Pennsylvania State University, Graduate Assistant, Practicum Supervisor, Teaching Assistant

2009-2010 Clemson University, Research Assistant, Independent Living Coordinator, Clemson LIFE program

2003-2009 Charleston County School District, Charleston, SC, Special Education Teacher, Students with Learning Disabilities, Students with Autism